Hartry Field. *Saving Truth From Paradox.* Oxford: Oxford University Press 2008. Pp. 406. (paperback ISBN-13: 978-0-19-923074-7).
Anita Burdman Feferman & Solomon Feferman. *Alfred Tarski*. Life and Logic. Cambridge: Cambridge University Press 2008. Pp. 425. (paperback ISBN-13: 978-0-521-71401-3).

Alfred Tarksi considered himself to be 'the greatest living sane logician', expressing simultaneously 'supreme confidence in his talent' (1) as well as a criticism of life-long challenge Gödel (who developed strange habits like wearing a mask). Tarski is mostly known by philosophers for his work in semantics (justifying the study of truth in a formal fashion) and *Tarski's Theorem* (that languages with basic means of self-reference cannot contain their own truth predicate). His work, besides these classics, comprehended wide areas of logic. With others he invented meta-mathematics as the study of properties of formal systems themselves (e.g. decidability, soundness, independence of axioms…). He invented decision procedures, number theories, types of algebras – and a lot more. After the Second World War he made the University of California into the world's centre for logic. Famous logicians (like Dana Scott or Richard Montague) studied logic with Tarski.

Tarki's biography by Anita and Solomon Feferman is now available in a paperback edition. They write from the perspective of former Tarski students. Students knew Tarski from late night session of carefully re-working single phrases of publications, Tarski, himself staying awake on coffee and amphetamines, urging them on. The Fefermans euphemistically describe a person others may consider an egocentric megalomaniac. Some of the behaviors they describe as 'a life-long need for women' (cf. pp. 158, 178, 196, 200…) nowadays would be filed under 'sexual harassment' (of students). What is more interesting about *Alfred Tarski* is less the admiration for the person of the great logician one may share or not, but the insightful view into the early days of analytic semantic theory (before the Second World War) and Tarski's empire building in logic (after the Second World War). Even the story of a logic genius shows itself to depend on many chance events. Most dramatically Tarski only left on the eve of the Second World War to tour the USA. Had he stayed he, converted to Catholicism, but being of Jewish descent (originally named 'Teitelbaum'), most certainly would have been killed like many other Polish logicians, famous nowadays for single theorems (like Lindenbaum or Presburger) as they were murdered by the German occupants. For the whole war he had to fear for his family, his wife and children surviving, other family members and colleagues being killed.

The Fefermans not only picture the biography of Tarksi, but also set out, in six 'Interludes' beside the biographic narration, some of Tarki's major achievements and areas of work. Thus students and readers interested in the history of analytic philosophy and logic, and being vaguely familiar with the areas Tarki's name is associated with certainly benefit from *Alfred Tarki: Life and Logic*.

Tarski's treatment of the notion of truth and its paradoxes superseded the syntax centrism and hostility to semantic concepts that prevailed in the Vienna Circle up to Tarski's "The Concept of Truth in Formalized Languages" in 1935, including Carnap's just published *Logical Syntax of Language* (1933). Carnap devoted himself to semantics, and Tarski's work became classical. Tarski aimed at formalized languages only, as he took natural languages to be universal (i.e. including their own semantics) and thus inconsistent. His approach works by distinguishing the definition of truth in a meta-language L^+ from the object-language L for which 'true in L' is defined. A stratification of truth predicates enfolds. Although Tarski himself did not extend this idea to natural languages one may hope to treat (simple/natural) truth this way, but treatments of this type have been heavily criticized for several reasons (like the strict separation of levels of truth apparently absent in natural discourse, inexpressibility of the method itself if true etc.), so

that new paths to deal with the paradoxes have been explored.

Hartry Field has devoted much of his work in the last years to the study of the antinomies of truth and property theory. *Saving Truth from Paradox* provides both an overview on ways of dealing with the paradoxes of truth as well as an introduction to Field's own approach to save truth from paradox.

Field's approach is a version of a gap-approach (i.e. he denies *tertium non datur* [TND] for the problematic sentences like the Liar). Field uses several building blocks from other theories. Therefore the first part of the book introduces *inter alia* Kripke's fixed point construction for a theory of truth and Lukasiewicz's continuum valued logic. Field shares some of the criticism of Tarski-style stratified truth theories. He works, like Kripke's construction, with iteration instead of stratification: Starting with a ground-level of sentences not involving 'true' more and more sentences (i.e. now sentences involving 'true', speaking of other sentences involving 'true'…) are assigned to the positive extension of 'true sentence'. As there are only countable many sentences, somewhere (i.e. somewhere in the non-finite ordinals) the construction has to settle into a fixed point, delivering the ultimate extension of 'true sentence'. Kripke's own three-valued construction contains no conditional and ultimately has to fall back to stratification. Field therefore uses a three-valued or a continuum-valued logic in the fashion of Lukasiewicz. To avoid some pitfalls of Lukasiewicz's construction Field introduces a special conditional beside material implication. The conditional is true at a stage if there is an ordinal (in the preceding iteration process) starting from which the antecedent always has a lower semantic value than the consequent; false is starting from some ordinal it always has a higher semantic value; neither true nor false otherwise. This conditional has to be used where TND fails (i.e. in the critical semantic sentences); where TND holds it is identical to material implication. The logic of this conditional is, of course, weaker than standard propositional reasoning. Field overall theory makes heavy use of limit ordinal constructions. Field finally is able to derive his central result: His construction can conservatively extend a model of the semantics free ground language by evaluating all the truths evaluations, *and* do this by having *both* the Truth Scheme [True(A)↔A] and intersubstitutivity of 'True(A)' and 'A'. 'It is only insofar as the unsubscripted predicate "True" transcends the Tarskian hierarchy that it is nonclassical.' (275)

The justification of Field's approach depends crucially on a comparison to other approaches to the paradoxes. Field thus compares his approach to 'classical solutions' (one part of the book) and paraconsistent solutions (the final part of the book).

Classical solutions keep classical logic, and so have to give up at least one direction of the Truth Scheme. Either way they have to endorse bizarre claims: Giving up the left-to-right direction means having theorems saying that some sentence is true *without* having that sentence itself *or* even having its negation! Giving up the right-to-left direction means having some sentence as theorem *without* being able to say that the sentence is true *or* even saying that it is not true! *Saving Truth from Paradox* works meticulously through many filiations of such theories and provides a veritable field guide in that area. Such theories seem worse than giving up TND for some sentences.

Paraconsistent solutions keep the Truth Scheme, but change the underlying logic, just like Field's solution. In distinction to Field's 'paracomplete solution' which has some sentences being neither true nor false, a paraconsistent solution, at least dialetheism, may have some sentences being both true and false. Field tries to argue that paraconsistent solutions face worries worse than paracomplete solutions (*inter alia* problems of expressing determinate truth or falsity, extending the ubiquity of true contradictions to simple arithmetic…). It is not at all clear that these criticisms apply to paraconsistent solutions in general, as Field focuses more or the less

exclusively on Graham Priest's dialetheism and Priest's criticism of Field. There are several paradigms of paraconsistent logics (e.g. adaptive logics, which have interesting conditionals), which may be better positioned to answer Field's challenges and have a better net balance of virtues and vices than Field's solution. Whereas the part dealing with the classical solutions in itself recommends *Saving Truth from Paradox* the comparison with paraconsistent solutions is far from settled. Sometimes intuitions clash: Dialetheism denies intersubstitutivity of 'True(A)' and 'A' in the scope of negation, which Field challenges as counterintuitive, whereas Field subscribes to *verum ex quodlibet sequitur* [e.g. $A \rightarrow (B \rightarrow B)$], counterintuitive to Relevant Logics (one of the areas of paraconsistent logic).

Field himself considers some of the typical challenges to gap-theories. Beside his theory of truth he considers determinacy operators at length, constructing an additional theory of being determinately true (once again involving fixed points somewhere beyond some limit ordinal, where on pains of regaining the paradoxes the determinacy iteration must not collapse). Field believes this theory to be immune to revenge and almost free of counterintuitive drawbacks.

Notwithstanding the technical sophistication of his overall treatment of matters, this positive self-assessment needs further elaboration. For instance: Field makes short work of the problem that one might introduce exclusion negation again by a postulate '$\neg A$ is true iff A is not true'. If that worked one would have a negation with TND and thus regain paradox. Many gap-approaches have the problem that their meta-theory allows – on pains of losing the power to express some semantic fact – the re-introduction of exclusion negation, and thus of Strengthened Liars. Field rejects such a postulate as it works 'only if we assume Boolean laws for the "not" used in making the stipulation' (310). Nicely put, but unconvincing. Compare: You have three collections of items and operations of moving one item from one to the other; now the three collections are placed/distributed over a border; there are three ways to do this, in all cases one collection is opposite to the others (making now the across the border region); there is an operation of moving an item from one of the opposing two collection to this collection. This is perfectly structurally isomorphic to having three collections of sentences, divided in the true sentences, the false sentences and the gappy sentences. As one can introduce a border with collections of marbles (the green vs. the non-green) it is possible to have a border between the only true sentences and the other two collections. The operation across the border is exclusive negation. Thus either a Strengthened Liar is re-introduced (bad for Field) or although the semantics is isomorphic to the marble model the semantic fact of a (possible) border cannot be expressed: expressive limitations (also bad for Field). *Except*, the structural analogy between marbles and sentences gives in – but this needs some heavy duty metaphysical work, not yet delivered by Field.

Further on, Field proves a lot of theorems about fixed points and limit ordinals, i.e. levels of iteration that we finite beings certainly do not 'reason up to in stepwise fashion'. We can, of course, prove theorems about these infinite ordinals. What about the reasoning about these limit stages and fixed points – where does it take place? Field often distinguishes truth from truth in a model and validity (for some semantics in some model) from genuine validity, for which then proving soundness seemed bared by *Gödel's Second Incompleteness Theorem* (cf. 45-49). If such reasoning is not feasible according to Field's theory we express something inexpressible, thus mystery. If answering these concerns one supposes to talk at the ultimate fixed point stage it doesn't sound that way: What about the usage of 'true' at this stage? It seems we are at a level very like the first semantic level above the ground language, but there is no where to iterate to anymore to avoid paradox (as we have, by assumption, exhausted all countable ordinals)! If this is a classical meta-language, we are back to Tarskian stratification and nothing is gained! Field answer to that challenge (which could be put as asking for the truth theory for the set theory ZF

used in the model theory) that 'we have an adequate truth theory for ZF_{true} within ZF_{true}' (356). He explicitly promises in the introduction 'that there are languages that are sufficiently powerful to serve as their own meta-languages' (18). But his construction contains its theory only in that sense of 'theory' that the set of theorems containing 'true' is included within it. The meta-theorems he proves are of another kind. They speak about the whole hierarchy. Field has to be more explicit about the status of his meta-theory and its resources. This is especially pressing in his treatment of determinate truth. He sees the problem that with the determinacy operators (i.e. the operators 'it is determinately true that…' for any amount of iteration) we do not have the same construction as with 'true', where we have only iteration. We have the idea of a 'super-determincacy' operator claiming something to be true *tout court*. Finally Field seems to yield to expressive limitations: 'the claim that I dispute is that the model theory *ought* to allow for super-determinateness operator meeting intuitive preconceptions' (357). The meta-theorems he puts forward, I gather, are meant to be super-determinately true, but he denies that they can be so.

Saving Truth from Paradox is a challenging book. The reader has to have advanced background knowledge and understanding in meta-logic and semantics. The treatment is at times Byzantine but most times exiting.

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