Mark van Atten. ***Essays on Gödel's Reception of Leibniz, Husserl and Brouwer*****. Logic, Epistemology, and the Unity of Science, Vol, 35, Springer, 2015, 327pp.**

Van Atten's book is a detective story on Gödel's philosophy of mathematics.
Gödel has the status of a kind of cult figure in at least popular science accounts of the history of logic in the 20th century. His reclusive way of life and mental disorder (leading to his self-starvation) have added to this picture. People might thus be interested in his greater philosophy of mathematics beyond his more technical papers. And in deed this might be interesting for the academic philosophy of mathematics as well, as one might suspect that such an important logician might have important ideas to contribute to this field. Expanding on Gödel's ideas in the philosophy of mathematics (like his emphasis on some form of Platonism and mathematical 'intuition') might open new options within the field. Unfortunately Gödel did not publish any philosophical essay on the philosophy of mathematics. Neither has he left any finished manuscript. He discussed philosophy of mathematics with quite a number of people, made notes, and (versions) of some of his published papers contain notes hinting at his philosophy of mathematics. Van Atten interprets these pieces of evidence and tries to found a reading of Gödel's philosophy of mathematics on them. As they are sparse and underdeveloped van Atten interprets the same short remark or thrown in sentence over and over again in the book. Pieces of the puzzle are added by reference to memories some people have of conversation with Gödel, and referring to passages (e.g. in Husserl's books) that Gödel read and might have made use of or agreed to.

The main body of the book is divided into three parts, each referring to one of the philosophers Gödel reflected upon: Leibniz, Husserl, and Brouwer. The book consists mostly of previously published papers by van Atten, thus one finds considerable overlap and repetition. Gödel's formal work is not discussed in detail.

With respect to Leibniz Gödel tried to read into Leibniz account of monads a metaphysics resembling his own ideas. The focus of interest being twofold here: on the one hand the idea of objectivity being guaranteed by concepts being in God's mind, and on the other hand the idea of reflection. Reflection as a principle of set theory roughly says that if some structural condition is true for sets then it is true in a part of the set theoretical universe (i.e. one can have an example without access to the whole set theoretical universe). The importance of the principle resides as well in its equivalence to the Axiom of Replacement, which is needed to guarantee the existence of sets of higher infinite cardinality. Gödel toyed with an analogy to Leibniz's idea of reflection between the single monad and the relations within the universe of monads. Taken as an argument one cannot proceed from Leibniz' metaphysics to any specific statement of mathematical structural truth, van Atten shows.

(Gödel also showed interest in Leibniz's notion of reductive proof, which inspired more than one logician as a model of building definitions or proofs on a sound basis.)

Gödel, moreover, considered Leibniz's account of the subjective consciousness of the monad and its access to knowledge as underdeveloped, and it is here that he turned (in the 1950s) to Husserl. Husserl's phenomenology provides, as Gödel at least remarked to several people, an approach capable of solving the problem of intuitive access to mathematical (categorical) entities. Phenomenological descriptions may elucidate how we intuit concepts like 'possibility'. Husserl's notion of intuition fits better to Gödel's agenda than Kant's notion of intuition and its role in mathematics, since Husserl aims at the essence of intuition, a form of intuition shared by any mind whatsoever. If a phenomenology of this type succeeded it would intuit mathematical objects as they are given in the mind of God, and existence in His mind guarantees – this being Gödel's tenet – their objectivity. In contract to Husserl, who speaks of categorical intuition of individual mathematical entities, Gödel is focussed on our intuitive grasp of concepts (like the concept of 'powerset'). If our grasp of the concept of powerset can be secured then we have secured all its applications, especially its role in generating non countable infinities and the Continuum. Gödel thus aims at justifying the axioms (of set theory), an approach that we find today, for instance, in George Boolos' work on the iterative hierarchy.

Brouwer, in distinction to Gödel's attempt at appropriating parts of Leibniz' and Husserl's philosophy, served for Gödel as a contrast and a challenge to his view in the philosophy of mathematics. They express contrasting ideas of mathematical reality and the very worth of mathematics, which Brouwer at times derided as aberration of pure subjective thought, whereas Gödel revered mathematics as our access to the absolute. Both share some mystical and illuminational tendencies.

The centre piece of this part of the book is an essay on Gödel's 'Dialectica-interpretation' of intuitionism (so called because it appeared in the journal *Dialectica*). The Dialectica-interpretation brings together Gödel's reflection on intuitionism and his approval of relying on some form of (phenomenological) intuition of basic concepts. The interpretation is founded on the concept of a 'computable function of finite type' to be extensible in elucidation (i.e. not fixed in a formalism or mechanical algorithm). Our grasp of this concept is taken to be revealed by *a priori* psychology (this being Brouwer's intuitionism in Gödel's eyes) or something like phenomenological psychology. Given this foundation Gödel and Brouwer share the rejection of a mechanization of mind (ala Turing), but Gödel, of course, claims our grasp of further concepts, way beyond what basic computable functions are. Even Gödel's reading and interpretation of intuitionism are not the intended ones by Brouwer. Gödel substitutes his notion of 'reductive proof' (going back to definitions, somewhat in the way of Leibniz) for the intuitionist's general reference to 'proofs', taken by Brouwer to be based in individual mental acts.

Thus in the main part of the book we learn how Gödel dealt with parts of Leibniz' and Husserl's philosophy, and how he tried to partially reconcile or deal with Brouwer's intuitionism as an alternative philosophy of mathematics. This belongs to an intellectual biography of Gödel, more so than setting out any new contribution to phenomenology by Gödel. We see no new arguments in Gödel's philosophy of mathematics beyond the known desiderata. Gödel praises phenomenology and hints at phenomenology in his reflection on intuitionism, but detailed phenomenological analyses are missing. Also his reference to Leibniz is more an analogy than a new foundational argument.
One can after reading this part understand the somewhat surprising fact that Gödel – typically associated with modern formal logic and part time member of the Vienna Circle – turned to Husserl. That even Gödel could not make substantial progress from Husserl to the philosophical foundations of set theory may justify that one should not expect further contributions to a realist philosophy of mathematics from that direction.

The last part of the book contains a systematic essay in which van Atten defends Brouwer against Gödel on Husserl's ground: Brouwer and Husserl share many of their foundational thoughts. Some of Brouwer's claims can best be understood within Husserl's phenomenology. This applies also to specific theses like a restriction of mathematics to the potentially infinite only.
If von Atten is right on this, and he sets out a strong case for it, then the combination of Gödel's ideas and phenomenology was still born. If Husserl and Brouwer see mathematical objects as constructions, this in the end limits their approach to some form of constructive mathematics, and then Gödel's turn to Husserl must have been in vain.

Students of Gödel thus may find in von Atten's book interconnections between Gödel's scattered remarks on the philosophy of mathematics, but no unified Gödelian philosophy of mathematics. This could have been put forth in one longer comprehensive essay, in itself much shorter than the collection of essays.

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