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Universal Logic

- There is no simple summary to all of paraconsistent logic, since there are so many paraconsistent logical systems, divergent approaches and applications. There is, however, an interesting question that may arise at least for a dialetheist: Are we to be logical relativists, choosing a different system for each different purpose, or shouldn't we – in the spirit of semantic closure, so to speak – look out for a system that is able to deal with all these purposes.
- Such a system would be a *universal* logic.
- For philosophy one might wish for a univeral logic. Philosophy was taken here as a field of univeral inquiry/reflection. In case universality is only possible with a severly restricted logic, however, all philosophical reasoning in the decisive fields – including this introduction – had to be recast in that logic, where many standard arguments simply do not go through. So on the other hand the univeral logic in question should be *adaptive* to distinguish consistent from paraconsistent contexts. Standard logic should simply be correct – not only semi-correct (Priest 1987, pp.137-53) – in consistent contexts.

Weak Universal Logic

- A universal logic might be universal as a paraconsistent logic, i.e. in all fields in which we need *a paraconsistent* logic this logic can be employed and gives acceptable results. This may be called *the weak universalist program*.
- One may take the weak universalist program as being extreme precacious: One takes one's favoured paraconsistent logics – and sticks to it in *all* contexts. Since this paraconsistent logic can deal with contradictory contexts it can deal with any context, so it really is universally applicable.
- The problem with this extreme caution is that one loses all otherwise available consequences in consistent contexts. Therefore one rather tried to distinguish the type of context one is reasoning in. In praxis this meant that we employ standard First Order Logic for all non-semantic or non-antinomic contexts and switch to paraconsistency only in our formalization of complete semantics (or, maybe, set theory).

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Strong Universal Logic

- Or a truly universal logic can be employed everywhere, supposedly containing a way to distinguigh consistent from inconsistent contexts, without loss of proper logical power in comparison to FOL. This may be called *the strong universalist program*.
- In case philosophy contains consistent contexts and uses arguments valid only in consistent contexts it seems to need to follow either a strong universalist or a corresponding adaptive program.[cf. Chap.7]
- Both the LFI-approach and Adaptive Logics follow the idea to be able *within the system used* to distinguish contexts of a stronger logic (usually FOL) and contexts for a paraconsistent logic. The way they do it is completely different, however. In the LFI-approach the distinction what kind of context we have has to be given *beforehand*; only given the corresponding knowledge can we choose the appropriate formalization (i.e. use °A or not); in Adaptive Logics we mark the supposition that some formula has to be consistent, a supposition that may be *revised* in the process of reasoning; no prior knowledge about the consistency bahaviour of a context is required.

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Universal Logic 1

- Richard Routley has in his (1979a) an appendix with the nice title "Ultralogic as Universal". He defines: "A universal logic, in the intended sense, is one which is applicable in every situation whether realised or, possible or not." So this idea is tied to question of modality and inconsistent ontology – and Routley's *noneism* [see Chap. 17].
- Standard logic should be recaptured in those areas where it is valid, according to Routley; he keeps *Tertium Non Datur*, but claims that universal logic should allow for truth value gaps.
- He proposes the Relevant system **DKQ** the first degree structures (i.e. without nested entailments) of which are those of the Relevant logic **RQ** (i.e. a quantified version of system **R**).
  [Remember from Chap. 5 that **R** contains all **PC**-theorems, for "⊃". *Modus Ponens* is only given for "→".]
- The semantics of **RQ** needs something like the Routley Star and the ternary accessibility relation; **DKQ** has, according to Routely, no finite characteristic truth tables for its sentential part!
- Given the last point one may better look elsewhere for universal logic.

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Universal Logic 2

- Brady's Logic $\mathbf{DJ^{d}Q}$ [cf. Chap. 5] is announced as a universal logic.
- Brady explicitly says that the aim should be to have a universal logic and not chose one's logic for each department of research. Standard reasoning should be recaptured in consistent contexts, again.
- In $\mathbf{DJ^{d}Q}$ neither Disjunctive Syllogism, nor Absorption, nor the *Modus Ponens*-Theorem (for "$\rightarrow$") nor *Tertium Non Datur* are valid! So Brady introduces "classical sentences" (p', q' ...) for which Disjunctive Syllogism and *Tertium Non Datur* should hold.
- Brady uses his system not just to avoid trivialization from antinomies, but to avoid the antinomies in the first place by the invalidity of *TND*! (The Liar and its negation are *both* taken as false, against the spirit of Convention (T) and what the Liar says, it seems.)
- In this – two sorted – formal system the power of standard logic is indeed recaptured, but in the process of *formalization* we have to know already which sentences (or predicates) *are* "classical", i.e. consistent!
- Negation Introduction may be a problem since in $\mathbf{DJ^{d}Q}$ not even:

$$(A \rightarrow \neg A) \rightarrow \neg A.$$

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Universal Logic 3

- Jean-Yves Beziau works on a programme he calls "universal logic"; but in his case the title is not meant as referring to some specific logical system, but to a study of the most universal features that logics have. Beziau's "universal logic" is rather a *general theory* of logics.
- The key question is that enterprise is: "What have all logical systems in common?" or even simpler "What is logic?"
- Beziau sees the common ground in the *abstract* idea of a *consequence relation* (abstract in the sense of not being specified by certain axioms or rules) and the abstract notion of a *non-trivial maximal set*, the set being maximal with respect to a consequence relation; non-triviality being the more universal notion than consistency (as in maximally consistent sets in standard model theory), as shown by PLs.
- [Note: It is most unfortunate that the label "Universal Logic" thus came to denote two quite different approaches. Beziau (2005, p. viii) claims that he "coined"/invented the term "universal logic", but Routley used the term at least 10 years earlier for his approach!]

Universal Logic 3 (II)

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- The study of logics in this generality comprises the study of abstract/non-specific proof theory.
- The general study of logics is even more general than the study of algebraic logical structures.
- In this *abstract theory of logic* one may – especially if one frees oneselves from any scruples about only technically motivated systems – define in the very abstract what a logic is.
- Tarski introduced three basic conditions on a consequence relation:
  1. Identity/Reflexivity: $A \vdash A$
  2. Monotinicity/Weakening: $A \vdash B$, then $A, C \vdash B$
  3. Transitivity/Cut: $A \vdash B$ and $B \vdash C$, then $A \vdash C$.
- Even the most deviant logics that one comes across fulfil usually these requirements. Default Logics, being non-monotonic, of course, fail condition (2).

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Universal Logic 3 (III)

- One may even give a more general definition of a logic by requiring only reflexivity and transitivity of "$\vdash$". One may then define:

  A logic is a preorder on the Cartesian Product of the power set of a set of sentences.

  The consequence relation as partial order that is reflexive and transitive relates sets of sentences to other sets of sentences (if we even allow multiple conclusions). [cf. (Straßburger 2005)]
- Theorems – seen thus – have an empty premise set. Inconsistent premise sets have – in some logics – no consequences at all.
- This very abstract definition of a logic abstracts away from proof theory, which may be questionable.
- It also allows for infinite premise sets, which are – at least more often than not – beyond *our* logical capacities.

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More Universal Logics

- One may take one's favourite LFI-system or adaptive logic system as a universal logic. Both approaches have it as part of their official doctrine that it should be possible to recapture standard reasoning (remember daCosta's original idea, [Chap. 8]).
- Proponents of the corresponding camps within paraconsistency (like Diderik Batens) are, however, outspoken logical particularists, i.e. they propose that one choses a logic given a particular task or topic at hand.
- We will not pick some particular of these systems here, since – as we have seen in Chap. 7 and Chap. 8 – both approaches have virtues and shortcomings in comparison to versions of **LP**.
- So let us try a little mixing of ideas! As a little summary overview, and as a statement to a supposed universal logic, let us develop here a universal logic as a system of natural deduction with adaptive features and extended with operators of modalities and inconsistency with some relative of **LP** as the lower limit logic.

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Universal Natural Deduction

- The adaptive idea corresponds to the dynamic character of our knowledge concerning the consistency of some context; it also incorporates the *bona fide* epistemic principle that given no other information we should by default reason with the strongest available logic.
- Natural deduction stays closer to the actual process of deriving consequences from premises than axiomatic systems, not to speak of trees and other *ex post* decision procedures (where you test a specific formalization of a specific supposed consequence).
- Natural deduction – although redundantly – can often characterize the connectives by introduction and elimination rules that give the meaning of the connectives. It may be supplemented by axioms (rules with empty antecedents) and definitions.
- For ease of application we develop a mixed system here, proposing rules for basic connectives, but also having some definitions.

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Universal Natural Deduction (II)

- We take the format of natural deduction shown in Chap.7: We number the line, then include in "< >" the number of *assumptions* this line depends on, then give the formula, then the operation by which this line was generated, then the set of formulas *presupposed* to be *true* (to apply the restricted rules, see below), and if needed the marked individual constants for the quantifier rules.
- We built this system by using the following ingredients:
  - an adaptive version of standard **PC** (cf. Minimal Inconsistent LP);
  - quantifier rules in the vain of *Minimal Free Description Theory* (allowing for the use of descriptions, including those that are – as are some individual constants – non-referring);
  - identity rules that restrict substitution to consistent objects;
  - rules for a stronger conditional, whether we really need this or not;
  - basic rules for modalities.
- The system thus is dynamic and maybe incomplete; but it is a truly universal logic incorporating the basic ideas we have been going through in this book.

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Universal Logic 4

- We could call the logic "Adaptive Extended LP with Minmal Free Description Theory, restricted Identity, Basic Entailment and S5 Modalities" or "**ALPeMFDrIES5**" for short, or just **UL4**.
- The definition of the language and the built up of derivations are as usual, with the modifications mentioned above.
- A typical derivation looks similar to the one's already given in Chap.7.
- The following slides give the rules for the connectives/quantifiers used in **UL4**. One may doubt whether they are all needed – as one may doubt whether one really needs modal logic or a modal entailment if there is an adaptive conditional already – but in any case we invest **UL4** with some extra expressive power, in case applications arise.
- As mentioned, we do not follow the strict format of natural deduction in giving *only* the meaning of all connectives – as far as possible – in introduction and elimination rules. The benefit of using **LP** as our fall back paraconsistent logic is that all standard tautologies are **LP** valid. So we can simply include all **PC** tautologies in **UL4**!

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Universal Logic 4 (II)

- **UL4** thus keeps one of the basic ideas of **LP** [respectively of Hallden's "Logic of Nonsense"]: keeping theorems and consequences apart. We have a lot – in the propositional case all – of the standard theorems, but that does not mean that one can detach (such) conditionals in general. *Reasoning* thus is paraconsistent and to some extent Relevant, whereas *logical truth* need not be.
- A benefit of this approach is that we keep besides the introduction and elimination rules of the connectives a "real" semantics with truth conditions for complex formula. A universally quantified conditional as used in science thus still can be understood as asserting something about reality, and not as a hidden statement on assertability.
- The semantics thus kept preserves the intuitive undestanding of the connectives and quantifiers we have seen, for example, in **LPQ**$^{=}$.
- Given this semantics one can take building blocks from the standard correctness proves to prove the correctness of the **UL4**-rules with respect to this semantics.

UL4 – Theorems and Assumptions

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- A line that reads

  n.< >   A

  contains a *theorem*, since the sentence A does not depend on any assumption (the dependency set noted within "< >" is empty). Theorems can be introduced into derivations at any time. [Letters "n", "m" etc. are used to refer to unspecified line numbers. Remember: "A" is a schematic letter, the object language having sentences like "p", "F(a)" etc.]
- To include **PC**-tautologies, which we know already, we have the rule:

  (PC)   n.< >   A   PC $\varnothing$

  where A is any **PC**-theorem. The column with markings is empty.
- For any other theorems (i.e. already proven **UL4**-theorems) we have:

  (TH)   n.< >   A   TH Γ

  where A is any **UL4**-theorem. Γ contains the presuppositions. There cannot be a list of marked individual constants in theorems.
- To introduce *assumptions* into a derivation we have the rule:

  (AE)   n.<n>   A   AE $\varnothing$

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UL4 – ∧

- Conjunction Introduction has the form:

n.<m>	A	...	Γ
o.<k>	B	...	Λ
p.<m,k>	A ∧ B	(∧I) n, o	Γ ∪ Λ

- Conjunction Elimination has the two forms:

n.<m>	A ∧ B	...	Γ
o.<m>	A	(∧E) n	Γ

n.<m>	A ∧ B	...	Γ
o.<m>	B	(∧E) n	Γ

[Note: Here and in the following rules "<m>" refers to an unspecific (number) of assumptions that the line depends on. Γ can, of course, be empty; if there are marked individual constants they are marked only in the line where the quantificational rule is employed, see below.]

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UL4 – $\vee$

- Disjunction Introduction has the two forms:

n.<m>	A	...	Γ
o.<m>	$A \vee B$	$(\vee I)$ n	Γ

n.<m>	A	...	Γ
o.<m>	$B \vee A$	$(\vee I)$ n	Γ

- Disjunction Elimination has the form:

n.<m>	$A \vee B$	...	Γ
o.<k>	$\neg A$	...	Λ
p.<m,k>	B	$(\vee E)$ n,o	$\Gamma \cup \Lambda \cup \{°A\}$

This is the restricted form of Disjunctive Syllogism.

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UL4 – ¬

- Negation Introduction has the form:

n.<n>	A	AE	∅
o.<m,n>	¬A	...	Γ
p.<m>	¬A	(¬I) $\underline{n}$,o	Γ

If some assumption allows to derive its own negation, then this sentence can be stated negated *simpliciter* (i.e. the status as assumption is discharged, as indicated by the underlining in the line using (¬I)). The usual form of Negation Introduction leads to trivialization in inconsistent contexts.

- Negation Elimination has the form:

n.<m>	¬¬A	...	Γ
o.<m>	A	(¬E) n	Γ

This double negation elimination is anti-intuitionistic.

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UL4 – ⊃

- Conditional Introduction (*Conditionalization*) has the form:

n.<n>	A	AE	$\varnothing$
o.<m,n>	B	...	Γ
p.<m>	$A \supset B$	$(\supset I)$ $\underline{n}$,o	Γ

This rule mirrors the Deduction Theorem, which is valid both for **PC** and **LP**. If the conditionalization is the last step of a derivation the restrictions on not having marked individual constants in it have to be kept.

- Conditional Elimination (*Modus Ponens*) has the form:

n.<m>	$A \supset B$	...	Γ
o.<k>	A	...	Λ
p.<m,k>	B	$(\supset E)$ n,o	$\Gamma \cup \Lambda \cup \{°A\}$

This is the restricted form of *Modus Ponens* corresponding to the restricted Disjunctive Syllogism. We have, of course:

$$\vdash_{UL4} (A \supset B) \equiv (\neg A \vee B)$$

UL4 – Truth, Falsity, Inconsistency

- The truth operators used in Chap. 4 and 8 can be introduced as well, in a syntactically extended language. Truth Introduction/Elimination follow the disquotational (T)-schema. Strict Truth/Falsity will be defined notions. The Inconsistency operator is treated by rules as well.
- Truth Introduction has the form:

n.<m>	A	...	Γ
o.<m>	TA	(TI) n	Γ

- Truth Elimination is the converse:

n.<m>	TA	...	Γ
o.<m>	A	(TE) n	Γ

- Falsity Introduction has the form:

n.<m>	¬A	...	Γ
o.<m>	FA	(FI) n	Γ

- Falsity Elimination is the converse:

n.<m>	FA	...	Γ
o.<m>	¬A	(FE) n	Γ

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UL4 – Truth, Falsity, Inconsistency (II)

- Inconsistency Introduction has the form:

n.<m>	$A \wedge \neg A$	...	Γ
o.<m>	$^{\bullet}A$	($^{\bullet}$I) n	Γ

- Inconsistency Elimination is the converse:

n.<m>	$^{\bullet}A$	...	Γ
o.<m>	$A \wedge \neg A \wedge \neg{}^{\circ}A$	($^{\bullet}$E) n	Γ

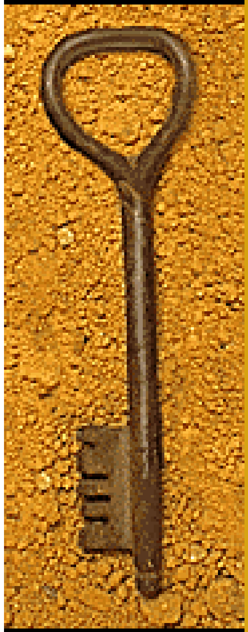
These two rules follow the idea that an "•" formula is a hidden conjunction [cf. Chap. 8].

- For simple truth we introduce its version of Convention (T):

n.<>	$\Delta A \equiv A$	(Δ)	$\varnothing$

This works since detachment [as ($\supset$E) or ($\equiv$E)] and Contraposition are restricted to consistent sentences. Defining ΔA as $TA \wedge \neg FA$ will not do, since $TA \wedge \neg FA$ is true for antinomies! Restricting rules for "T" or "F" would leave the truth or falsity of antinomies inexpressible.

- The consistency operator "°" is taken as defined notion then.



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UL4 – □

- Necessity Introduction (*Necessitation*) has the form:

n.< >	A	...	Γ
o.< >	□A	(□I) n	Γ

A theorem (but not any sentence depending on further assumptions) can be necessitated, like in *normal* modal logics.

- Necessity Elimination has the form:

n.<m>	□A	...	Γ
o.<m>	A	...	Γ

What is necessary is also the case. This corresponds to the basic axiom of standard modal logic **T**: $\Box A \supset A$.

UL4 – □ (II)

- Since necessity is taken here to be semantic necessity (not natural necessity or some more restricted version of necessity) it has to governed in the way of a normal modal logic of the strength of modal system **S5**. Therefore we need two further rules:
- The rule corresponding to the K-Axiom of modal logic has the form:

n.<m>	$\Box(A \supset B)$	...	Γ
o.<k >	$\Box A \supset \Box B$	(K) n,o	Γ

Note that employing *Modus Ponens* now with □A requires □A to be consistent (otherwise provable contradictions would lead to triviality).

- The rule corresponding to the **S5-**Axiom has the form:

n.<m>	$\Diamond A$	...	Γ
o.<m>	$\Box\Diamond A$	(S5) n	Γ

For (S5) it is not required that ◇A is a theorem.

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UL4 – →

- One may consider having also an entailment connective "→". Whether one needs one is another question. The reason "→" was introduced in the first part of this book was either that whereas principles failed for "⊃" one liked to have a conditional for which they hold, or that too Irrelevant consequences hold for "⊃". The latter reason has no force in the context of discussing universality: The more can be inferred the better may be the chances to capture consequences universally. The former reason is weakened by the adaptive approach: The principles we like to be valid sometimes are valid in these circumstances. So the *rationale* for having a further conditional connective is not clear.
- Further on, the semantics considered for "→" were mostly – with the possible exception of Brady's semantics of *contents* – not very convincing. The ternary accessibility relation may be sometimes/ somewhat understood in information terms, but meaning relations are not adequately pictured thus.
- Once one has a modal semantics for "→", these modal frames should also provide the semantics for modal operators like "□".

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UL4 – $\rightarrow$ (II)

- Even **SKP** semantics is problematic then, since it [cf. Chap. 6] has to use a non-reflexive accessibility relation. If the modal accessibility relation is non-reflexive $\Box A \supset A$ is not valid, although it is one of the most intuitive principles concerning necessity. If the modal accessibility relation was reflexive, Absorption would not be invalid in **SKP**, however!
- One could solves this problem by either distinguishing several modal accessibility relations between worlds (one for each modal basic notion) or by taking the conditional in the problematic cases of Curry like paradoxes to be "$\supset$" instead of "$\rightarrow$", so that *Modus Ponens* is not in general valid and trivilization is blocked [cf. Chap. 3]. Both options are not satisfactory.
- Thus one may add some weak principles of entailment (like natural deduction versions of the sequent rules of **SKP**) *or* – and that is the approach I favour – one takes entailment like in standard modal logic to be *semantic entailment* in the sense that: $A \rightarrow B \Leftrightarrow \Box(A \supset B)$.

p ∧¬p

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UL4 – $\rightarrow$ (III)

- Taking entailment to be *semantic entailment* in the sense that:

$$A \rightarrow B \Leftrightarrow_{UL4} \Box(A \supset B)$$

gives us *derived* introduction and elimination rules for "$\rightarrow$".

- Entailment Introduction is a strict form of Conditionalization:

$n.<n>$	A	AE	$\varnothing$
$o.<o>$	B	AE	$\varnothing$
...			
$r.<n,o>$	C	...	Γ
$s.< >$	$A \wedge B \rightarrow C$	$(\rightarrow I)$ $\underline{n}$,$\underline{o}$,r	Γ

In strict conditionalization *all* assumptions have to be conditionalized (thus we get a *theorem* to be necessitated to yield the entailment). Of course, if there only is one assumption we may get $A \rightarrow C$.

- Entailment Elimination is a version of *Modus Ponens*:

$n.<m>$	$A \rightarrow C$	...	Γ
$o.<k>$	A	...	Λ
$p.<m,k>$	C	$(\rightarrow E)$ n,o	$\Gamma \cup \Lambda \cup \{°A\}$

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UL4 – $\rightarrow$ (IV)

- Since Entailment Elimination is a version of *Modus Ponens*:

n.<m>	$A \rightarrow C$	...	Γ
o.<k>	A	...	Λ
p.<m,k>	C	$(\rightarrow E)$ n,o	$\Gamma \cup \Lambda \cup \{°A\}$

  it is not valid in inconsistent contexts, so there is not a universally valid version of *Modus Ponens*, even if semantic entailment is around.
- This might be a reason to take "$\rightarrow$" not to be semantic entailment, but a Relevant conditional, characterized by the principles of some Relevant Logic, such that *Modus Ponens* holds universally – even in inconsistent contexts. This would cover the intuition that there are no exceptions at all to *Modus Ponens*.

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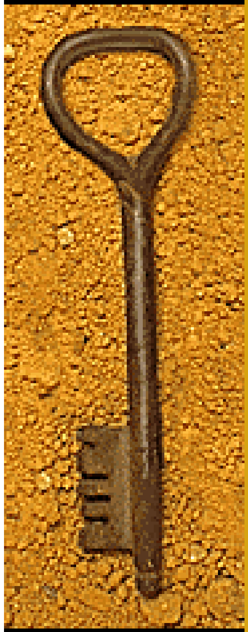
UL4 – Relevance

- If you look at those rules of **UL4** that discharge assumptions [i.e. introduce a sentence into the antecedent of a material or strict conditional or introduce a negation, the rules: (¬I), (⊃I), (→I)], then you will notice that the assumption(s) to be discharged *have to be in the dependency set* of the line where we employ these rules. This means that on the way to this line somewhere these assumptions really have been used.
- So **UL4** contains – besides the invalidity of *ex contradictione quodlibet* – another element of Relevant Logics. A simple introduction of an Irrelevant antecedent is not allowed.
- It is still possible, of course, to have Irrelevant antecedent *parts* introduced by assuming not just the sentence A needed for the derivation, but a conjunction $A \wedge B$ with some arbitrary other sentence. Then, however, the Irrelevant part is displayed in the derived sentence and not hidden somewhere in the derivation.

[Note that general transitivity and substitution of proven equivalents do not hold in **UL4**, thus the classical working around the restriction is not available.]



UL4 – Defined Connectives

- We introduce some further connectives just by definitions. There are, of course, derivable introduction and elimination rules then.
- Within a derivation we use the definitions by referring to their name:

(D≡) $A \equiv B := (A \supset B) \wedge (B \supset A)$

(D↔) $A \leftrightarrow B := (A \rightarrow B) \wedge (B \rightarrow A)$

(D◇) $\Diamond A := \neg \Box \neg A$

(D∇) $\nabla A := \Delta \neg A$

(D°) $^\circ A := \Delta A \vee \nabla A$

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UL4 – Quantification

- We use a quantificational logic in the manner of Minimal Free Description Theory. This version of Free Logic maintains that we may use individual expression – especially descriptions – that do not refer.
- Given these non-referring expressions the quantificational have to be revised, since one can only generalize from an arbitrary *existing* object or specify to an arbitrary objects if one knows that this object exists.
- The objects are divided into the mere *possible objects* and the *existing objects*. The existing objects give the domain of a given possible world.
- One may consider the broader domain as comprising just names and descriptions, and the existence claim as saying that some name or description is instantiated, thus avoiding the ontologically controversial *possibilia*, but this is a debate independent from paraconsistency, so let us just follow the usual modelling here.

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UL4 – Quantification (II)

- We have to give the usual requirements on marking individual terms in case of applying Universal Generalization or Existential Specialization within a derivation [cf. for example (Quine 1974), (Essler/Martinez 1991)].
- These are:
  - terms generalized in Universal Generalization and specialized to in Existential Specialization are *marked* at the *right* of such a line;
  - the marking also notes the dependencies on other individual terms in that line (in the form "a(e)" "a" being marked depended on "e");
  - markings may not be circular (i.e. we do not have "a(e)" *and* "e(a)");
  - no term may be marked twice;
  - marked terms may neither occur in the premises nor in the conclusion of a supposed valid derivation.
- In applications of the quantifier rules one also has to meet the requirement that by generalising one constant to a variable "x", "x" will not be bound by already otherwise existing quantifiers. (∀I) and (∃I) require further on that "x" and the individual term occur at *exactly* the same places in a given sentence.

$p \wedge \neg p$

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UL4 – Existence

- E!(á) says that the object denoted by á exists, "E!( )" being the existence predicate. (Of course it is a logical constant.)
- Quantifiers refer to existings objects only.
- "$(\exists x)F(x)$" really means there is an *existing* objects that is F.
- As **FOL** does implicitly – by demanding that the domain is not empty – here we also assume that there *is* something:

  $n.\langle\ \rangle \qquad (\exists x)E!(x) \qquad (E!) \qquad \varnothing$

[Note: There are versions of Free Logic (so called "Universally Free Logics") that do not assume that anything exists whatever – so making logic free of any assumptions of existence – but they have hardly any interesting applications, of course.]

[Note: Semantically speaking the extension of "E!( )" is the domain *at* a world index. Since quantifiers range only over existent objects here "E!( )" could be defined:

(DE!) $\qquad E!(a) := (\exists x)(x = a)$

the rule then being: $\qquad n.\langle\ \rangle \qquad (\exists x,y)(x = y) \qquad (E!) \qquad \varnothing$

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UL4 – =

- Identity Introduction is valid for any object, existing or not:

n.< >	$a = a$	(=I)	$\varnothing$

- Identity Elimination (i.e. substitution of identicals) is more critical. It has to be restricted to avoid trivilization in a paraconsistent logic with as much expressive power as **UL4**.
- We have to presuppose that some object is not an inconsistent object to apply (=E) to it. We define a consistency predicate "K( )" for objects (as a logical constant, of course) to do this:

  (DK) $K(á) := \neg(\exists P)(P(á) \wedge \neg P(á))$

  UL4 is no 2nd order system, but we may employ (DK) in that way that we note $\neg K(á)$ in some line if for the object named á we could have a line with an instance of the scheme: $P(á) \wedge \neg P(á)$.
- Identity Elimination then takes the form:

n.<m>	$P(á)$	...	Γ
o.<k>	$á = é$	...	Λ
p.<m,k>	$P(é)$	(=E) n,o	$\Gamma \cup \Lambda \cup \{K(é)\}$

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UL4 – = (II)

- Since we want to use description and modal operators we have to provide (=E) with a *provisio* in case descriptions are involved.
- In modal logic **S5** all modalities can be reduced to modalities of degree 1 (i.e. all modal operators except the most right ones canbe dropped)[cf. Hughes/Cresswell 1990, pp. 51-54].
- We require as a *provisio* for Identity Elimination:

  In case we have á = é, then

  (i) if á is a description and é an individual constant, é cannot be substituted into a modal context of "$\Diamond$",

  (ii) if á is an individual constant and é a description, é cannot be substituted into a modal context of "$\Box$".
- Thus we avoid the typical counterexamples to modal (=E):

  (1) The-number-of-planetes = 9

  (2) $\Box(9 > 7)$

  (3) $\Diamond$(The-number-of-planets > 9)

  (*) $\Box$(The-number-of-planets > 7)

  (**) $\Diamond(9 > 9)$

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UL4 – $\forall$

- The following quantifier rules require following the rules of marking the constant generalized/specialized in $(\forall I)$ and $(\exists E)$, and the renaming of variables mentioned before.
- $\forall$-Introduction (Universal Generalization) has the form:

n.<m>	R(á,é)	...	Γ	
o.<m>	$(\forall x)$R(x,é)	$(\forall I)$,n	$\Gamma \cup \{E!(á)\}$	á(é)

Thus the application of $(\forall I)$ requires an existence assumption concerning á, since we conclude to a generalization about all *existing* objects. á is marked, here as depending on é.

- $\forall$-Elimination (Universal Instantiation) has the form:

n.<m>	$(\forall x)P(x)$	...	Γ
o.<m>	P(é)	$(\forall E)$,n	$\Gamma \cup \{E!(é)\}$

Since the generalization is (maybe) true of existing objects only the application of $(\forall E)$ presupposes that the constant specialized to names an existing object.

p ∧¬p

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UL4 – ∃

- ∃-Introduction (Existential Generalization) has the form:

n.<m>	P(á)	...	Γ
o.<m>	(∃x)P(x)	(∃I),n	Γ ∪ {E!(á)}

  Thus the application of (∃I) requires an existence assumption concerning á, since we conclude to a generalization about some *existing* objects.

- ∃-Elimination (Existential Instantiation) has the form:

n.<m>	(∃x)R(x,á)	...	Γ
o.<m>	R(é,á)	(∃E),n	Γ ∪ {E!(é)} é(á)

  Since the generalization is (maybe) true of existing objects only the application of (∃E) presupposes that the constant specialized to names an existing object. The name of the object is marked in its dependencies in the formula in question.

p ∧¬p

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UL4 – Existence Cancellation

- In case that existence assumptions are *explicitly* made the existence *presupposition* can be cancelled:

-

n.<m>	P(á)	...	Γ ∪ {E!(á)}
o.<o>	E!(á)	AE	
q.<m,o>	P(á)	(E!C),n,o	Γ

- If the existence claim follows from the other assumptions the presupposition can be cancelled as well:

n.<m>	P(á)	...	Γ ∪ {E!(á)}
o.<m>	E!(á)	…	Γ ∪ {E!(á)}
q.<m>	P(á)	(E!C),n,o	Γ

UL4 – Descriptions

- Minimal Free Description theory allows that there are non-referring names and descriptions. It assume that there is something [see (E!)]. It requires the uniqueness of a description with respect to the existing objects only – so to say: if the thing does not exist, the question of its uniqueness does not arise. Otherwise it looks like the standard Russellian account of descriptions.
- We use the usual "ι"-notation, so that "ιxF(x)" means "*the* (unique) F".
- The (MFD)-rule can be stated as the following two ways of interchangability:

n.<m>	$\iota x P(x) = \acute{a}$	...	Γ
o.<m>	$(\forall y)(\acute{a}=y \equiv P(y) \wedge (\forall z)(P(z) \supset z=y)$	(MFD),n	Γ

n.<m>	$(\forall y)(\acute{a}=y \equiv P(y) \wedge (\forall z)(P(z) \supset z=y)$	...	Γ
o.<m>	$\iota x P(x) = \acute{a}$	(MFD),n	Γ

The first conjunct in the equivalence states satisfaction of the defining property, the second expresses uniqueness.

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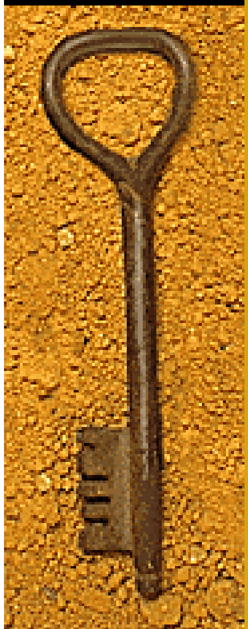

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UL4 – Presuppositions and Dab

- In the context of quantificational rules we can now make clear the reference to a set of *presuppositions* above.
- Adaptive Logics speak of Dab-formula and corresponding sets of consistency assumptions. In **UL4** we note these consistency assumptions as presuppositions to employ some restricted rules.
- Actually the consistency presupposition is "°A".
- In Minimal Free Description Theory usually a conjunct "E!(a)" is needed (e.g. as derivable line or *assumption*) to employ one of the quantifier rules. Since **UL4** is a dynamic logic already we need not work with "E!(a)" as a line in a derivation, but can note this also as a presupposition in the presupposition set Γ noted on the right. (This save us some work within the derivation and makes them more similar to standard ones. Existence presuppositions are dynamic!)
- In case of Identity Elimination the presupposition is that we have a consistent object. We note this as the presupposition "K(a)" for an object a in question.

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UL4 – Presuppositions and Dab (II)

- Each of the sentences in the presupposition set has a negation. Once the negation of such a presupposition can be derived, all lines are retracted which depend on that presupposition (like in the original adaptive dynamics).
- The retraction thus does not only concern the disappointment of consistency assumptions (either for a sentence or an object), but also the disappointment of existence presuppositions.
- If the last line A of a derivation has a non-empty presupposition set Γ, this means that the sentence in that line is derivable from the assumptions noted within "< >" given these further presuppositions.
- Let Φ be the (possibly empty) set of assumptions and Γ the (possibly empty) set of presuppositions. We have:

$$\neg(\exists B \in \Gamma)\ \Phi \vdash_{\mathbf{UL4}} \neg B \Rightarrow \Phi \vdash_{\mathbf{UL4}} A$$

- To save labour and have derivation looking more closely like standard derivations we adopt the convention to drop noting Γ if Γ is empty.

p ∧¬p

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UL4 – Examples

- Let us see some examples of **UL4**-derivations!
-

1.<1>	$G(\iota xF(x))$	AE	
2.<2>	$a = \iota xF(x)$	AE	
3.<2>	$G(a)$	(=E)	{K(a)}
4.<1>	$a = \iota xF(x) \supset G(a)$	$(\supset I)\underline{2},3$	{K(a)}
5.<>	$G(\iota xF(x)) \supset (a = \iota xF(x) \supset G(a))$	$(\supset I)\underline{1},4$	{K(a)}

- Descriptions carry no existence assumptions themselves, thus:

1.<1>	$a = \iota xF(x)$	AE	
2.<1>	$(\forall y)(a=y \equiv F(y) \wedge (\forall z)(F(z) \supset z=y)$	MFD,1	
3.<1>	$a=a \equiv F(a) \wedge (\forall z)(F(z) \supset z=a)$	$(\forall E)2$	{E!(a)}
4.<>	$a=a$	(=I)	
5.<1>	$F(a) \wedge (\forall z)(F(z) \supset z=a)$	$(\supset E)3,4$	{E!(a),°(a=a)}
6.<1>	$F(a)$	$(\wedge E)5$	{E!(a),°(a=a)}
7.<1>	$(\exists x)F(x)$	$(\exists E)6$	{E!(a),°(a=a)}
8.<>	$a = \iota xF(x) \supset (\exists x)F(x)$	$(\supset E)\underline{1},7$	{E!(a),°(a=a)}

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UL4 – Examples (II)

1.<1>	p	AE
2.<>	$p \supset \neg\neg p$	PC
3.<1>	$\neg\neg p$	$(\supset E)1,2\ \{°p\}$
4.<2>	$\neg p \vee q$	AE
5.<1,2>	q	$(\vee E)3,4\ \{°p, °\neg\neg p\}$
6.<2>	$p \supset q$	$(\supset I)\underline{1},5\ \{°p, °\neg\neg p\}$
7.<>	$(\neg p \vee q) \supset (p \supset q)$	$(\supset I)\underline{2},6\ \{°p, °\neg\neg p\}$

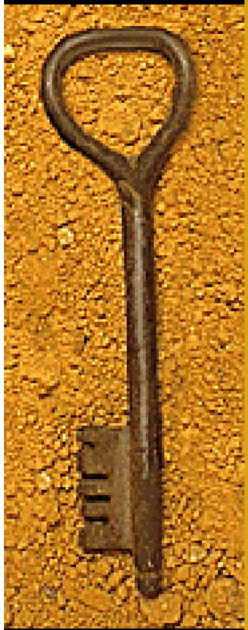
Proving something this way doesn't make too much sense, since one could have (7) by **PC**, detachment requires "$°(\neg p \vee q)$" anyway.

- So that some sentence is provable given some assumptions relative to some presuppositions does not mean that there is no other derivation which does not make so many presuppositions.

UL4 – Examples (III)

1.$\langle\rangle$ $\Diamond\neg p \supset \Box\Diamond\neg p$	(S5)($\supset$I)	
2.$\langle\rangle$ $\neg\Box\Diamond\neg p \supset \neg\Diamond\neg p$	(PC)($\supset$E)1	$\{°(\Diamond\neg p \supset \Box\Diamond\neg p)\}$
3.$\langle\rangle$ $\Diamond\Box p \supset \Box p$	(D$\Diamond$)2	$\{°(\Diamond\neg p \supset \Box\Diamond\neg p)\}$
4.$\langle\rangle$ $\Box p \supset \Diamond\Box p$	(TH)	
5.$\langle\rangle$ $\Box p \supset \Box\Box p$	($\supset$E)($\supset$I)3,4	$\{°\Box p, °(\Diamond\neg p \supset \Box\Diamond\neg p)\}$
6.$\langle 6\rangle$ $\Box p$	AE	
7.$\langle 6\rangle$ p	($\Box$E)6	
8.$\langle 6\rangle$ $p \vee q$	($\vee$I)7	
9.$\langle\ \rangle$ $\Box p \supset p \vee q$	($\supset$I)$\underline{6}$,8	
10.$\langle\ \rangle$ $\Box(\Box p \supset p \vee q)$	($\Box$I)9	
11.$\langle\rangle$ $\Box\Box p \supset \Box(p \vee q)$	(K)10	
12.$\langle\rangle$ $\Box p \supset \Box(p \vee q)$	($\supset$E)($\supset$I) 5,11	$\{°\Box p, °(\Diamond\neg p \supset \Box\Diamond\neg p)\}$

"($\supset$E)($\supset$I)" abbreviates an application of *transitivity* (with a temporary assumption to the antecedent of the first conditional). The restrictions on *Modus Ponens* may – like in this example – introduce a long list of presuppositions, sometimes even requiring consistency of *theorems*.



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UL4 – Consequence

- Consequence in **UL4** may be defined:

  ($\vDash_1$) $\Gamma \vDash_{UL4} A$ iff

  in case that all $B \in \Gamma$ are true at least, then A is true at least.

- Nothing needs to be said concerning the case that any $B \in \Gamma$ is false *only*. One has not to hold that then a consequence relationship holds. To do so would be Irrelevant.
- To do so may come close to reintroducing Explosion, as well. ΔA and ∇A are incompatible, so both can never be true at the same time, so allowing for Irrelevant consequences would yield, for example:

  (**) $\nabla A, TA \vDash_{UL4} C$

  for any C.
- To insist that the "in case" has to be read as material implication as in **PC** just begs the questions against a Relevant meta-theory!

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UL4 – Consequence (II)

- An improved Relevant definition of consequence in **UL4** thus might be:

  $(\vDash_2)$ $\Gamma \vDash_{UL4} A$ iff there are models such that all $B \in \Gamma$ are true at least, and in case that all $B \in \Gamma$ are true at least in a model, then A is true at least in that model.

- The existence condition rules out the Irrelevant cases and (**).
  A consequence relation obtains if and only if all of the non-empty set of models that make the premises at least true make the consequence at least true.
  As always $\vDash$ concerns the inheritance of truth.
- This version, $(\vDash_2)$, requires then some reworking of the proof theory.

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UL4 – Consequence (III)

- Changing the definition of consequence this way requires a further book keeping of presuppositions, in this case with respect to assumptions.
- In as much as **UL4** has to be *correct* the basic rules must not support consequence claims that go against the definition ($\vDash$) above. Making an unsatisfiable assumption, however, would allow claims like

  (*) $\nabla A \wedge TA \vDash_{UL4} \nabla A \wedge TA$

  (**) $\nabla A \wedge TA \vDash_{UL4} \nabla A$

  where the premise set is unsatisfiable and thus the claims are incorrect.
- Doing nothing would make almost any interesting consequence relation invalid, even ($\wedge$E) as in (**). Clearly, however, the solution is straight forward: assumptions (i.e. claims to be considered for further consequences) are presupposed not to be true, but to be *satisfiable*.
- When applying the **assumption rule** (**AE**) we have to use the form

  n.<n>   A   AE   sat({A})

  where we define the satisfiability presuppositions by the schema

  (sat) sat(Γ) $\equiv$ Γ has a **UL4**-model

  Γ being a set of assumptions.

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UL4 – Consequence (IV)

- The set of assumptions Γ has to be *jointly satisfiable*.
- With sat({A}) we note only the satisfiability of an *individual* assumption. If a line depends on several assumtions, the further assumptions entering into its derivation also have to enter the set the satisfiability of which is presupposed.
- The presupposition of satisfiability is cancelled when Γ contains or entails for some A either

  (i) $TA \wedge \nabla A$ or (ii) $\nabla A \wedge \Delta A$ or (iii) $\Delta A \wedge FA$
- In case the presupposition later turns out to be violated lines depending on the assumption in question have to be retracted (as always).
- Since we *generally* have to presuppose the satisfiability of the set of assumptions which a line depends on, we may use the convention of not especially noting this in ordinary cases.

UL4 – Any Liars Around?

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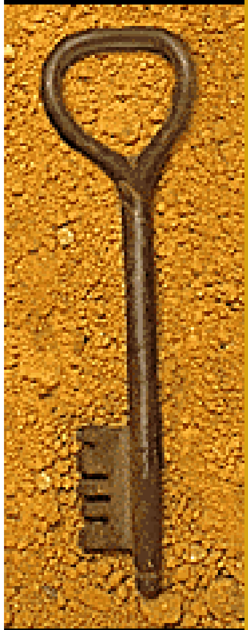
- ΔA and ∇A are incompatible, so both can never be true at the same time, so one may ask oneself whether some sort of Strengthened Liar may not re-introduce some sort of Explosion, like for any C:
  $$\nabla A, TA \vDash C$$
- Suppose a Liar like: (λ) $\nabla\lambda$
  Luckily the following derivation is *not* **UL4** valid:

1.	$T\lambda \vee \neg T\lambda$	(PC)
2.	$T\lambda \supset \lambda$	Convention (T)
3.	$\neg T\lambda \supset \nabla\lambda$	Definition "∇"
4.	$\nabla\lambda \wedge T\lambda$	1,2,3, Convention (T), Definition λ

- Interestingly the standard arguments for the standard Liars do not go through in **UL4**, the Liar forcing a logic on us which undercuts the derivation of the Liar although room was made for it to be derivable!
- This may result in the situation that a Liar like the one introduced above tries to say of itself that it is false only, but it does not succeed!

Dialetheism Reformulated



- Dialetheism can be weakened to the thesis that *if* given some basic principles of truth, denotation, membership and (semantic) closure we derive at contradictions, these may be taken as being true. If some well known examples are lost that does not matter.
- The *purpose* of dialetheism is not to have true contradictions, but to have semantic closure or naïve set theory or … even if this involves accepting some true contradictions. The controversy has centered on the dialetheist's claim that there are true contradictions, but the starting point has always been some other philosophical tenet. So in case there are no true contradictions, so the better for semantic closure or naïve set theory.
- I take the philosophical point of dialetheism to be that even if there are true contradictions this is the price to pay in some universal theory/logic. So we better model logic, reasoning, belief and assertion on the assumption that there are true contradictions. I just assume here – in the spirit of the Framework Hypothesis of chapter 2 – that some Liar (still) can be proven and thus is taken as true by a dialetheist.

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UL4 – Assessment

- **UL4** has several merits:

1. It is paraconsistent, correct and non-trivial.
2. It may model inconsistent ontology – if needed.
3. It provides two conditionals to distinguish truth conditional reasoning from contrafactual reasoning or expressing sufficient and necessary conditions by entailments.
4. It obeys Convention (T) with disqoutational rules.
5. It expresses (in)consistency within the language – if needed.
6. It is as standard a logic as possible without trivialization, so fulfils the methodological Minimal Damage criteria of chapter 3.
7. It does not validate *Modus Ponens* in general, so avoids Curry's Paradox; it shouldn't, given the arguments reconsidering the Modus Ponens Condition, dropping the object-/meta-language distinction; transitivity also isn't validated in general.
8. It does not validate Contraposition in general, so dialetheism is not a dialetheia itself; moreover, by the binary truth-value operators semantic evaluations can be expressed as being true only.

UL4 – Assessment (II)

9. It restricts substitution of identicals, so can avoid hypercontradictions; this also blocks the excessive usage of inconsistent objects in dialethic set theory using **UL4.**
10. Replacement of provably equivalents [$A \dashv\vdash B$] holds, but not replacement of proven equivalents [$\vdash A \equiv B$], as the later would also blur the identities of true contradictions.
11. **UL4** shows Relevance in discharging only used assumptions.

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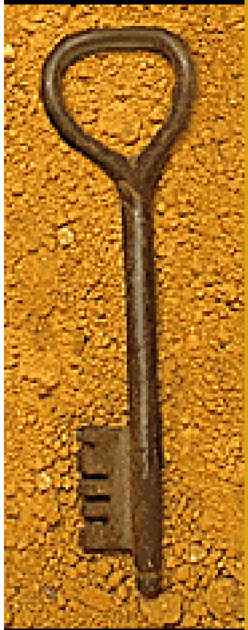
UL4 – Assessment (III)

- The main shortcoming of **UL4** – as an adaptive logic – is that its provability relation isn't recursive enumerable, because of its dynamic.
- More precisely, *final* derivability isn't recursive enumerable. The *relative derivability statements* (i.e. those statements like

$$\vdash_{UL4} G(\iota x F(x)) \supset (a = \iota x F(x) \supset G(a)) \qquad \textit{given } \{K(a)\}$$

  expressing that something is derivable from a (empty) set of premises on the given set of presuppositions) *are* recursive enumerable. So one should not exaggerate the failure of enumerability of theorems!
- From a more general perspective one may say that if we restrict our interests to the finite or the simple propositional structures, then we have decidability; if we proceed to the infinite or quantificational or more complex propositional structures, we lose decidability but keep recursive enumerability of theorems; if we proceed further to universality (no longer requiring consistency), then we lose also recursive enumerability of theorems. Still we have a logic to reason with and the relative derivability statements are enumerable and correct.

UL4 – Assessment (IV)



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- The *relative derivability statements* with respect to logical consequence (i.e. derivability from a set of assumptions) now carry the presupposition that the assumptions/premises are satisfiable (in the defined sense above):

$$p \wedge q \vdash_{UL4} p \qquad \textit{given} \ \mathrm{sat}\{(p \wedge q)\}$$

expressing that something is derivable from a satisfiable set of premises.

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Universal Logic 5

- **UL4** is still a logic developed from an abstract point of view. "Universal Logic", however, might also be understood as the logic that *we* really use (that is we *humans* as *finite* beings using natural languages to *communicate* with each other and to *coordinate* our behaviour). "really use" is meant here as the (philosophical, but ultimately also psychological) claim that somewhere in our mind there is just this logic.
- The universal logic in that fifth sense would be the core of the capacities that are operational not only when we are explicitly reasoning, but also when we understand or translate language.
- This sense of universality takes up the old idea of trancendental philosophy that there is an ultimate structure of our conceptual scheme, a structure that all human languages share; possession of which we exhibit, for example, in interpreting some new behaviour as display of communication [cf. (Bremer 200y)].

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Universal Logic 5 (II)

- Universal Logic in that sense deals only with some of the logics that are considered logics by the abstract logic approach (**UL3**). Logics with infinitary premise sets or non-compositional semantics are not up for us.
- Universal logic in that sense would be Kant's Trancendental Logic in new clothes. **UL1**, **UL2**, **UL4** may strive to be part of that! Some requirements of rationality – like closure conditions – may have to be restricted. This calls for a Wide Reflective Equilibrium in the study of rationality [cf. (Stein 1996)].
- What this universal logic is may be hard to find out, but it has to be there.

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Questions

- (Q1) Why does the usual form of Negation Introduction lead to trivialization in inconsistent contexts? The rule is:

n.<n>	A	...	$\emptyset$
o.<m,n>	$\neg B \wedge B$	...	Γ
p.<m>	$\neg A$	$(\neg I)$ $\underline{n}$,o	Γ

- (Q2) Why do we have to assume that A is consistent in Entailment Elimination?
- (Q3) Why can't we have the LFI-style definition of "°"?

  (D°) $°A := \neg{}^\bullet A$
- (Q4) Why can we have replacement of provably equivalents, but not replacement of proven equivalents [$\vdash A \equiv B$]?
- (Q5) Even though "a=a" we note the presupposition of its consistency "°a=a" if required by some rule. Why can't we drop this in case even of theorems?

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Exercises

- (Ex1) State the introduction and elimination rules corresponding to (D$\equiv$), (DΔ), (D∇) and (D$\leftrightarrow$).
- (Ex2) Why does the Liar reasoning concerning $\nabla\lambda$ not go through in **UL4**?
- (Ex3) Prove the sentence in the right column from the premises in the left. What presuppositions do you need?

$p \supset q, \neg q$	$\neg p$
$\Box p, \Box q$	$\Box(p \wedge q)$
$p \supset q$	$(q \supset r) \supset (p \supset r)$
$p \vee q, p \supset r, q \supset r$	r
	$(\forall x)E!(x)$
$(\forall x)F(x)$	$\neg(\exists x)\neg F(x)$
$(\forall x)(F(x) \supset G(x))$	$(\forall x)F(x) \supset (\forall x)G(x)$

$p \wedge \neg p$

Manuel Bremer
Centre for Logic, Language and Information

HEINRICH HEINE UNIVERSITÄT DÜSSELDORF

Further Reading

- On the Relevant Logics of Routley and Brady see Chap. 5.
- On Beziau's work see his (1995) and his self-description "From Paraconsistent Logic to Universal Logic", a version of which appeared in: *Sorites*, 12 (2001), pp. 12-32. Recent work in this area is collected in (Beziau 2005).
- One may see (Wolenski 1995) as an exercise in universalizing logic in that Wolenski argues that logic is not mainly concerned with inheriting truth but designated values even if the designated value of a "dual logic" was falsity.
- On Minimal Free Description Theory see the essays in (Lambert 1991); on the format of natural deduction used here and the interpretation of outer and inner domain see (Bremer 2000a); on Priest's own version of a Free Logic variant of **LP** see (Priest 1999).
- There was a "1st World Congress and School of Universal Logic" in 2005, see the website: http://www.uni-log.org .