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Semantics

- Corresponding to semantic closure being one of the main motivations of strong paraconsistency semantics is one of the important fields in which PLs have to be tested.
- The cornerstone is a theory of truth for a language L given within that language itself.
- A theory of truth that obeys convention (T).
- A theory of truth that not only gives the truth conditions for the non-semantic vocabulary of L, but also of the semantic vocabulary itself.
- The construction of a theory of meaning/truth of a language can follow the general procedure we know from Tarski's theory of truth (Tarski 1936) and Davidson's extension/re-interpretation of that procedure as a theory of meaning (Davidson 1967).

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Language

- The basis of the construction can be the language of **SKP** or **LP** (extended with some Relevant conditional).
- We take "$\rightarrow$" here as a conditional for which Contraposition holds. [We could avoid using Contraposition by working with more axioms (for every case where we make essential use of contraposition); but for ease of explanation we employ Contraposition.]
- Besides the semantic vocabulary we need arithmetical means of naming expressions.
- "SAT" denotes the relation of satisfaction between sequences of objects and well-formed formula.
- "SUC" denotes the successor function. SUC(1)=2. The successor of any non-number is that object itself.
- "NUMB" denotes the set of natural numbers.
- "FORM" denotes the set of well-formed formula.
- "TERM" denotes the set of singular terms of the language.
- "FUNC" denotes the set of functors of the language. "FUNC^n" denotes the set of n-ary functors. FUNC does *not* denote the set of *functions*.

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Language (II)

- "PRED" denotes the set of *predicators* of the language; resp. "PRED^n".
- "CONS" denotes a function which takes indices of constants (these indices being a subset of the natural numbers) to the set of constants, and which takes non-numbers to a specific but arbitrarily chosen dummy constant.
- "VAR" does the same that CONS does for variables.
- "ROW", abbreviated as "( )+( )" denotes the function that adds the second argument (a sequence) to the first argument (a sequence) building the longer combined sequence.
- "APP" denotes a binary function such that APP(x,y) gives the value of the function or sequence denoted by x at the denotation of y (resp. the value of applying the function denoted by x to the argument denoted by y); and if there is no such object gives a specific but arbitrarily chosen dummy object.
- "SUB" denotes the triadic function such that "SUB(x,y,z)" denotes the function that agrees with the function denoted by x except that its value at the argument denoted by y is the value of the argument denoted by z.

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Language (III)

- "DEN" denotes a binary function, such that the value of "DEN(x,y)" is the denotation of the expression y, given that the free variables in y have been given values by the sequence x.
  [Note: "DEN" does *not* denote the general function of denotation.]
- Variables can take as values not only objects of the domain, but also numbers, sequences of objects, and sentences.
  Since we want to talk about the relation between linguistic entities and their denotation we have to be able to quantify over both and put them into relation.
- Each object *o* is identified with its unary sequence <*o*>.
  This is done to yield a general notion of satisfaction of predicates in relation to sequences that covers also the case of unary predicates.
- If s is a sequence and x a number s(x) is the x^{th} object in the sequence s. "s(x/y)" says that in the sequence s y is put at the place of x.

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Naming

- In general the name of an expression is build by <u>underlining</u>.
- For every singular expression of the language we can define its name thus (N being the set of natural numbers):
  - $\underline{v}_n$ is VAR($\underline{n}$) for $n \in N$
  - $\underline{c}_k$ is CONS($\underline{k}$) for $k \in K \subseteq N$

  the variable function VAR applied to the name of a number (a numeral) yields the nth variable's name. The function CONS applied to a numeral yields the kth constant's name.
- Given this taken together with the names of the logical vocabulary and the predicates we can build for any sequence *s* of expressions the name $\underline{s}$ of this sequence by using the function ROW.
  $s_1 s_2 \ldots s_n$ being a sequence of expressions $\underline{s}_{\underline{1}}+\underline{s}_{\underline{2}}+\ldots+\underline{s}_{\underline{n}}$ being the name of that expression.
- Thus we can talk in L about every expression of that language, quantify over them and can apply predicates – especially semantic predicates – to them.

Axioms for Semantics

- To make the structural theory of meaning possible we need *axioms for semantics*. (Not to be confused with *meaning postulates* for the non-logical vocabulary of a specific language; the axioms for semantics are not all semantic axioms, but deal in general with the needed apparatus of semantics.)
- These axioms concern:
  - (i) mathematical functions
  - (ii) syntactic operations
  - (iii) axioms for the denotation relation DEN
  - (iv) axioms for the satisfaction relation SAT

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Axioms for Mathematical Functions

- (Axiom 1) a. $(\forall x)SUC(x) \neq 0$
  b. $(\forall x,y)(SUC(x)=SUC(y) \rightarrow x=y)$
  c. $NUMB(0)$
  d. $(\forall x)(NUMB(x) \rightarrow NUMB(SUC(x)))$

  Axiom 1 deals with the successor function in the usual way. Note that (1b) is true also for non-numbers.

- (Axiom 2) $(\forall x)(a = APP(s(x/a),x))$

  The value at x of a sequence that substitutes a for x is a.

- (Axiom 3) $(\forall x,y)(x \neq y \rightarrow APP(s,x)=APP(s(x/a),y))$

  If x and y are different the value of a sequence has not changed at y if the value at x has been changed.

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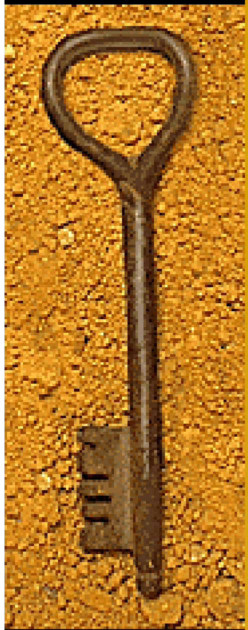
Axioms for Syntactic Operations

- (Axiom 4) $(\forall x,y,z)(x+(y+z))=((x+y)+z)$
  ROW is associative.
- (Axiom 5) a. $(\forall x)\mathrm{TERM}(\mathrm{VAR}(x))$
  b. $(\forall x)\mathrm{TERM}(\mathrm{CONS}(x))$
  c. $(\forall x_1 \ldots x_n, y)(\mathrm{TERM}(x_i) \wedge \mathrm{FUNC}^n(y) \rightarrow \mathrm{TERM}(y + x_1 + \ldots + x_n)$
  Variables and constants are terms, and a n-ary functor builds a new singular term out of the singular term arguments.
- (Axiom 6) a. $(\forall x_1 \ldots x_n, y)(\mathrm{TERM}(x_i) \wedge \mathrm{PRED}^n(y) \rightarrow \mathrm{FORM}(y + x_1 + \ldots + x_n)$
  b. $(\forall x,y)(\mathrm{FORM}(x) \wedge \mathrm{FORM}(y) \rightarrow \mathrm{FORM}(\,\underline{(} + x + \underline{\wedge} + y + \underline{)}\,) \wedge \mathrm{FORM}(\,\underline{(} + x + \underline{\rightarrow} + y + \underline{)}\,) \wedge \mathrm{FORM}(\,\underline{\neg} + x))$
  (6a) defines basic formula, (6b) complex formula by the basic connectives ("$\underline{\wedge}$" is the *name* of conjunction, and so on). (6c) correspondingly for quantification.
  c. $(\forall x,y)(\mathrm{FORM}(x) \wedge \mathrm{NUMB}(y) \rightarrow \mathrm{FORM}(\,\underline{(} + \underline{\forall} + \mathrm{VAR}(y) + \underline{)} + \underline{(} + x + \underline{)}\,)$



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Axioms for Denotation

- (Axiom 7) $(\forall x)(NUMB(x) \rightarrow DEN(s,VAR(x))=s(x))$
- (Axiom 8) $DEN(s,CONS(\underline{k}))=c_k$ for $k \in K$
- (Axiom 9) $(\forall x_1...x_n,y)(TERM(x_i) \wedge FUNC^n(y) \rightarrow$
  $DEN(y+ x_1+...+x_n)=y(DEN(s, x_1)...DEN(s, x_n))$

- These axioms define the denotation of terms recursively.
- The axiom corresponding to (Axiom 9) for formulas (i.e. with $PRED^n$) is no topic for DEN, but for SAT.

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Axioms for Satisfaction

- (Axiom 10) $(\forall x_1 \ldots x_n, y)(\mathrm{TERM}(x_i) \wedge \mathrm{PRED}^n(y) \rightarrow$
  $(\mathrm{SAT}(s, \mathrm{FORM}(y + x_1 + \ldots + x_n) \leftrightarrow$
  $y(\mathrm{DEN}(s, x_1) \ldots \mathrm{DEN}(s, x_n))))$

  A sequence s satisfies a formula iff the predicate applies to the terms (these maybe completed by s). This is the satisfaction condition for basic formulas. The further axioms deal with complex formula.
- (Axiom 11) $(\forall x, y)(\mathrm{FORM}(x) \wedge \mathrm{FORM}(y) \rightarrow$
  $(\mathrm{SAT}(s, \mathrm{FORM}(\,\underline{(} + x + \underline{\wedge} + y + \underline{)}\,) \leftrightarrow$
  $\mathrm{SAT}(s,x) \wedge \mathrm{SAT}(s,y)))$
- (Axiom 12) $(\forall x, y)(\mathrm{FORM}(x) \rightarrow (\mathrm{SAT}(s, \mathrm{FORM}(\,\underline{\neg} + x) \leftrightarrow$
  $\neg\, \mathrm{SAT}(s,x)))$
- (Axiom 13) $(\forall x, y)(\mathrm{FORM}(x) \wedge \mathrm{FORM}(y) \rightarrow$
  $(\mathrm{SAT}(s, \mathrm{FORM}(\,\underline{(} + x + \underline{\rightarrow} + y + \underline{)}\,) \leftrightarrow$
  $\mathrm{SAT}(s,x) \rightarrow \mathrm{SAT}(s,y)))$
- (Axiom 14) $(\forall x, y)(\mathrm{FORM}(x) \wedge \mathrm{NUMB}(y) \rightarrow$
  $(\mathrm{SAT}(s, \underline{(} + \underline{\forall} + \mathrm{VAR}(y) + \underline{)} + \underline{(} + x + \underline{)}\,) \leftrightarrow$
  $(\forall z)\mathrm{SAT}(s(y/z), x)))$

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Defining Truth

- Truth can be defined in Tarski's way:

  (DTRUE) $\qquad \text{True}(x) \leftrightarrow (\forall s)\text{SAT}(s,x)$

  A formula (nothing else could be a proper argument of SAT) is true if and only if it is satisfied by all sequences.
- Mirroring the usual proof one can derive at the following theorem:

  (THEOREM) For any closed formula x: $\vdash (x \leftrightarrow \text{True}(x))$

  stating the disquotational character of truth.

- This theorem is established by establishing a couple of Lemmas. By the naming convention and Axiom 5 we get:
  (Lemma 1) It t is a term of L, then $\vdash \text{TERM}(\underline{t})$

- Correspondingly we can establish by Axiom 6:
  (Lemma 2) If x is a formula of L, then $\vdash \text{FORM}(\underline{x})$

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Defining Truth (II)

- Say that for any term or formula x the *relativisation* of x to s (x_s) is obtained by replacing any free variable v_k in x by the k^{th} object in s.
- With Axioms 7 to Axiom 9 we can prove for denotation DEN:
  (Lemma 3) For any term t of L $\vdash$DEN(s,$\underline{t}$)=t_s
- We approach truth more closely with the *essential* lemma:
  (Lemma 4) For any formula x of L $\vdash$SAT(s,x) $\leftrightarrow$ x_s

Proof (Outline): By induction on the complexity of the formulas of L. For basic formulas we get by Axiom 10:

$\quad$ SAT(s,$\underline{P(t_1 ... t_n)}$) $\leftrightarrow$ P(DEN(s,$\underline{t_1}$)...DEN(s,$\underline{t_n}$)

By Lemma 3: SAT(s,$\underline{P(t_1 ... t_n)}$) $\leftrightarrow$ P($\underline{t}_{1s}$,...,$\underline{t}_{ns}$)

By Lemma 3: P($\underline{t}_{1s}$,...,$\underline{t}_{ns}$) $\leftrightarrow$ (P($\underline{t}_1$,...,$\underline{t}_n$))$_s$

In case of conjunctions: SAT(s,$\underline{x \wedge y}$) $\leftrightarrow$ ($x_s \wedge y_s$), ($x_s \wedge y_s$) $\leftrightarrow$ $(x \wedge y)_s$

Accordingly for negation and entailment; for quantifiers we get by Axiom 14 and the Lemmas 2 and 3 (renaming variables if necessary):

$\quad$ SAT(s, $\underline{(}$+$\underline{\exists}$+VAR($\underline{k}$)+$\underline{)}$+$\underline{x}$) $\leftrightarrow$ ($\exists$y)SAT(s($\underline{k}$/y),x)

x being the open formula, y being some object to replace v_k; by the IH

$\quad$ SAT(s, $\underline{(}$+$\underline{\exists}$+VAR($\underline{k}$)+$\underline{)}$+$\underline{x}$) $\leftrightarrow$ ($\exists$y)$x_{s(\underline{k}/y)}$

Filling in x other variables other than v_k, if any, provided by the objects in s, and using the variable y for v_k we get a open formula x' for which we have: ($\exists$y)SAT(s($\underline{k}$/y),x) $\leftrightarrow$ ($\exists$y)x', and by construction of x': ($\exists$y)x' $\leftrightarrow$ (($\exists$y) x)$_s$. ■

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Defining Truth (III)

- The scheme proved in (Lemma 4) is the satisfaction scheme:

  $\vdash SAT(s,x) \leftrightarrow x_s$

- A formula that does not contain free variables (i.e. a closed formula) is either satisfied by *all* sequences or by none. Being satisfied means in this case that

  $\vdash x_s$

  And since it is a closed formula, this comes to

  $\vdash x$

  since x does not contain any free variables.

- So by the *definition* of truth (DTRUE)

  $\vdash True(x) \leftrightarrow (\forall s)SAT(s,x)$

  the right hand side led by assumption (the closed formula being satisfied by all/one sequence) to the formula itself being the case, thus

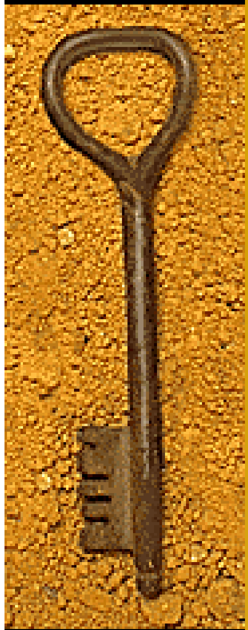
  $\vdash True(x) \leftrightarrow x$

  the essential (THEOREM).

Doing away with Contraposition

- As already mentioned our paraconsistent conditional will turn out to be one for which Contraposition does not hold. The corresponding logic (like **SKP**) will be justified in the light of our declared aims of strong paraconsistency only if we can do naive semantics with it, i.e. without Contraposition.
- This can in fact be done. We have to supplement the axioms of SAT with axioms for another relation ANTI-SAT, which does the work the negation of SAT does given the presence of Contraposition.
- The intuitive sense of ANTI-SAT is that ANTI-SAT(s,x) is true given some sequence s and a formula x iff that sequence makes the formula *false*.
- Instead of (Axiom 12) we take

  (Axiom 12') $(\forall x,y)(FORM(x) \rightarrow (SAT(s, FORM(\underline{\neg} + x)$
  $\leftrightarrow$ ANTI-SAT(s,x)))
- Further on we need axioms of making (complex) formula false. These axioms (15) – (17) are obvious for the connectives and quantifiers, given their standard treatment in the axioms (10) – (14).



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Doing away with Contraposition (II)

- For entailment we give

(Axiom 18) $(\forall x,y)(\mathrm{FORM}(x) \wedge \mathrm{FORM}(y) \rightarrow$
$(\text{ANTI-SAT}(s,\mathrm{FORM}(\underline{(} + x + \underline{\rightarrow} + y + \underline{)})) \leftrightarrow$
$\Diamond(\mathrm{SAT}(s,x) \wedge \text{ANTI-SAT}(s,y)))$

- In the light of **SKP** semantics an entailment is false if it is possible (there is a accessibility to worlds starting from the actual world @ such) that the antecedent of the conditional is true while the consequent is false. The consequent being false means that ANTI-SAT obtains.
- The proof of the essential (THEOREM) then has to be modified accordingly by substituting for each appeal to Contraposition an application of *Modus Ponens* to some instance of the axioms for ANTI-SAT.

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A Theory of Truth

- The construction outlined delivers a theory of truth (and correspondingly of meaning) of a language L in that very language.
- The theory satisfies that for every sentence of the language we get an instance of convention (T).
- "every sentence" means *every* sentence, including those dealing with semantic properties themselves.
- It gives a *systematic presentation of semantic closure*.
- Given the expressive power employed and semantic closure, of course the antinomies (or some versions of further antinomies) can be derived in this theory.
- This semantics is *inconsistent*.
- Given the non-triviality of the underlying logic (e.g. **SKP**), this semantics, nevertheless, is *paraconsistent*.
- Whether the extended theory for a fuller segment of a natural language is non-explosive depends on how many more axioms with how much logical power we add, for example, which axioms of arithmetic.

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Antinomies in this Theory of Truth

- The language structure given so far has not the means to formally prove the Liar or other antinomies by diagonalization or other arithmetic means.
- But since constants can *name sentences* we can form some Liar:

  $(\lambda) \qquad c = \underline{\neg True(c)}$

  Trying to satisfy c will lead us to $SAT(s,c) \leftrightarrow \neg SAT(s,c)$
- Replacing in our mathematical axioms the following axiom:

  (Axiom 1c) $\qquad NUMB(0)$

  and adding the axiom that any object is the x^{th} object of some sequence (which should be so given a domain and sequences over it):

  (Axiom 19) $\qquad (\forall x,y)(NUMB(x) \rightarrow (\exists s)(s(x)=y))$

  we can derive a contradiction for the SAT relation.

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Antinomies in this Theory of Truth (II)

- We define another version of SAT as SAT_1:

  (1) $SAT_1(x,y) \leftrightarrow (\exists s)(s(\underline{i})=x \wedge SAT(s,y))$

  [for some i and taking free variables to be universally quantified]

  (2) $SAT_1(x,\underline{y_k}) \rightarrow (\exists s)(s(\underline{i})=x \wedge SAT(s, \underline{y_k}))$

  [y_k being some formula with some free variable v_i free for x]

  (3) $SAT_1(x,\underline{y_k}) \rightarrow (\exists s)(s(\underline{i})=x \wedge y_k(v_i/s(\underline{i})))$

  [by the definition of satisfaction]

  (4) $SAT_1(x,\underline{y_k}) \rightarrow y_k(v_i/x)$

  [by conjunction elimination and substitution of identicals]

  (5) $NUMB(i)$

  (6) $y_k(v_i/x) \rightarrow (\exists s)(s(\underline{i})=x \wedge y_k(v_i/s(\underline{i})))$

  [by our new axioms (1c) and (19), reversing the steps to line (4)]

  (7) $y_k(v_i/x) \rightarrow SAT_1(x,\underline{y_k})$

  (8) $y_k(v_i/x) \leftrightarrow SAT_1(x,\underline{y_k})$ [(4) and (7)]

- Into supposedly harmless line (8) we substitute $\neg SAT_1(v_i, v_i)$ for y_k and instantiate x with the name of that sentence, so that we get:

  (9) $\neg SAT_1(\neg SAT_1(v_i, v_i), \neg SAT_1(v_i, v_i)) \leftrightarrow$
  $SAT_1(\neg SAT_1(v_i, v_i), \neg SAT_1(v_i, v_i))$

  an antinomy!

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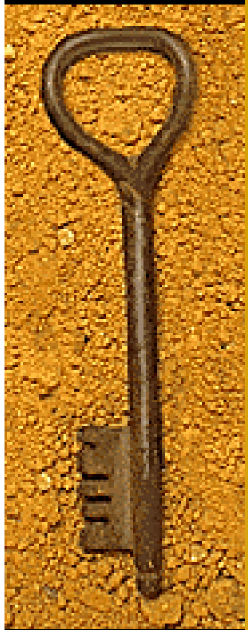
Antinomies of Denotation

- Denotation is one core concept of semantics. Self-referential antinomies like the Liar can be modelled in terms of denotation. Let DEN($\underline{x}$) be a monadic denotation function. For any closed term x, DEN($\underline{x}$) = x. (The denotation of *the name of* (the) x is x itself.)
- The *diagonalisation* of a term *a* substitutes *a*'s name ($\underline{a}$) for all free variables in *a*. Let the diagonal *function d* map names of terms to the *names* of their diagonalisation. $d(\underline{a(x)})=\underline{a(a)}$.
- Take an arbitrary term *b* with one free variable *x*. Consider *b(dx)),* let us call this *l*. The diagonalisation of *l* is $b(d(\underline{l}))$. Call this *s*. Since the diagonal of *l* is *s*, $d(\underline{l})=\underline{s}$. Substituting identicals within *b* we get $b(d(\underline{l}))=b(\underline{s}))$, but by the definition of s, this means $s=b(\underline{s}))$.
- Thus:

  (Theorem D1) For any term *b* with a free variable there is a closed term *s* such that $s=b(\underline{s}))$.
- Consider now *b(*DEN(x)*)*. By (Theorem D1) there is a closed term *s* such that *s*=*b(*DEN($\underline{s}$)*)*. Since *s* is a closed term, *s*=DEN($\underline{s}$). Thus:

  (Theorem D2) For any term *b* with a free variable there is a closed term *s* such that $s=b(s))$.

Antinomies of Denotation (II)



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- This means that there is a term that says/denotes the same as *some* function applied to this term. This gives as fixpoints for *all* functions: Points where the application of the function gives no further object.
- The mere existence of fixpoints is not problematic. *Being-identical-with( )* may be employed as often as you like to itself without resulting in a further object. The problem with the two theorems is that the expression *b* can be *any* functional expression of the language!
- Within a sufficiently rich language, however, these fixpoint lead into trouble.
- A fixpoint for the successor function SUC would be a number such that SUC(x)=x, i.e. a number that is its own successor, making arithmetic inconsistent.
- Hilbert and Bernays, who discovered the problems with denotation, concluded, therefore, that denotation cannot be represented *within* a consistent arithmetic itself (comparable to Tarski's Theorem concerning the representability of a truth predicate).

Antinomies of Denotation (III)

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- The resulting hierarchy is not acceptable for a dialetheist. At least a natural language as semantically closed should be able to represent denotation. So there has to be *some* arithmetic in which some number is identical to its successor. Thus we arrive at paraconsistent mathematics! [see Chap. 12]
- Nevertheless we need then some account of non-referring descriptions, because there is an expression available which is a fixpoint with respect to the parity function *f* (mapping even numbers to 1 and odd numbers to 0). This fixpoint s = *f*s . *f* maps 1 (being odd) to 0 and 0 (being even) to 1. If *f* is applicable to s then *f*s = 1 or *f*s = 0. Going through the cases gives us 1=0! Since triviality may ensue from this one may wish that s better not denote at all. [see Chap. 20]

Questions

- (Q1) What is the difference between denotation and satisfaction? What different interpretations of antinomies of denotation and satisfaction can be given in terms of the relation of language and reality?

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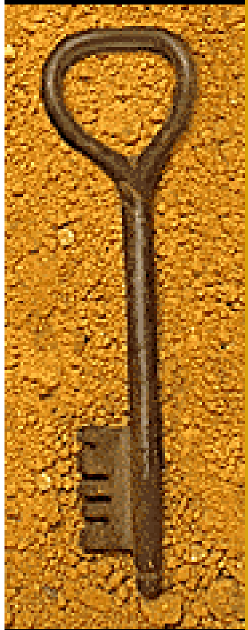

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Exercises

- (Ex1) Give the axioms for ANTI-SAT for conjunction, negation and the universal quantifier.
- (Ex2) Spell out in detail the reasoning concerning the fixpoint

  $$s = fs$$

  Does it go through in extended **LP**? How does it go in an adaptive version of **LP**?

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Further Reading

- The presentation is based on Priest, Graham. *In Contradiction*. A Study of the Transconsistent. Dordrecht, 1987, pp.157-77.
- On the denotation paradox by Hilbert and Bernays see: Priest, Graham. "On a Paradox of Hilbert and Bernays", *Journal of Philosophical Logic*, 1997, pp. 45-56.