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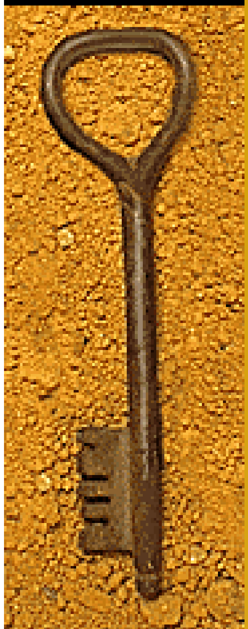
Epistemology and Philosophy of Science

- Dealing with and treating inconsistencies in theories as a topic of the philosophy of science is the fundamental justification of the weak paraconsistent approach.
- The use of a paraconsistent logic in that context is not only avoiding Explosion; the real confirmation of the thesis that scientists proceed with their theory development in the face of inconsistencies is a reconstruction of how this containment of inconsistencies and usage of inconsistent theories really is effected or can be effected.
- Supplementing the view on the actual workings of theory revision and updating the historical reconstruction of theory development and change as it really took place may foster the paraconsistent picture.
- Epistemic logic – as a branch of modal logic, like deontic logic – also has its own paradoxes. Deductive closure of belief has been considered paradoxical in epistemic logic. The theory of confirmation and probability theory contain the *Raven Paradox* or the *Lottery Paradox*. These are not antinomies, so it is not clear whether PLs have a job here, but there might be more severe cases!



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Inconsistent Theories

- Paraconsistent modelling can be employed diagnostically in case of historic examples of theories which are/were inconsistent, although they are/were not thought to be trivial or are/were not even known to be inconsistent.
- Examples given are:

  (a) the *classic calculus* of infintesimals in which the infinitely small amount are taken as *being nil* sometimes and as *being something* after all (one may divide through) at other times.
  [cf. (Moore 1990, chap. 4)]

  (b) *quantum mechanics* if you take the double slit experiments thus that some particle cannot pass both slits, but does so neverthe-less; but this commits you to an inconsistent ontology at the quantum level if you take this literally; weak paraconsistency might say that we only *talk* like that, but that this way of talking has to be superseeded.
  [A quantum logic giving up distributive laws (Putnam 1968) turned out to be not able to circumwent the problem (Gibbins 1981).]

Inconsistent Theories (II)

- (c) *Bohr's theory of the atom*, Bohr combing hypotheses *both* of classical electrodynamics with incompatible quantum hypotheses.

  (d) *Carnot's Theorem* in thermodynamics. The theorem was derived by Clausius in two ways using an inconsistent premise set, but he accepted the later proof, since it didn't use the inconsistency.

  (e) *Planck's Law*. Planck approached black body radiation is terms of *both* classical and quantum hypotheses.

  (d) *Newtonian Gravitation Theory*, since one for any nominated force on a test mass find a way of dividing the source masses of Newtonian cosmology so that we get the wanted resultant force.

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Inconsistent Theories (III)

- (e) *Standard Physics* is an interesting case because of the role mathematics plays here. According to the standard model of sub atomic processes there is a shortest non-zero interval of time (the ultimate quanta, which are not further divisible). The equations employed in standard physics, however, involve differentiation with respect to temporal coordinates, and a differentiation is only well defined there if it is possible to consider points/events that are *arbitrarily close* to each other. Thus the theory itself contradicts the mathematics that is otherwise employed to support or build up the theory! This does *not* compromise its predictive power, however!
  Thus either the theory is true *and* inconsistent or the real mathematics behind the theory is not the mathematics of the Reals!
  The reasoning from the applicability of mathematics in physics to the *truth* of its underlying assumptions (continuity in this case) is put into question *if* one wants to keep consistency.

  [This last option had important consequences for the maintainability of continuity assumptions in other fields – like the theory of computability and hypercomputability – as well.]

Working with Inconsistency

- The *historical thesis* behind these examples is that as long as there is no superseeding new theory or grounds to resolve the inconsistencies in a particular way, scientists will go on working with an inconsistent mixture of theory (elements), but avoid trivialisation.
- Thus the logic used here can only be paraconsistent!
- A *systematic thesis* behind these examples is that reasoning from the inconsistent mixture can serve as a heuristics towards the looked for consistent unified theory. (Lavoisier is cited as an example.)

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Realism or Instrumentalism?

- *Instrumentalism* is the view that the posits of science need not be taken at face value. Whether there are fifteen quarks or thirty-five is not of decisive importance, but whether the corresponding theoretical framework allows for successful predictions and technical applications in the field in question.
- From an instrumentalist perspective one may go as far as not caring whether the theoretical part of a science contains contradictions, as long as this is the best working theory; paraconsistent logic, in this case, preventing desastrous logical behaviour.
- From a *realist* perspective the contradictions posited by some theory which is held to be the best theory have to taken as true. In the case of empirical sciences this amounts to accepting inconsistent objects in reality! That seems just too much. [cf. Chap. 17 on inconsistent ontology]
- So one may be, although a realist in general, an instrumentalist about contradictions (thus advocating weak paraconsistency here). Without the realist attitude, however, there seems to be less motivation to solve the inconsistencies.

Inconsistent Observation

- The *basis* of empirical sciences is observation. Can there be inconsistent observations? [Note: "Observe" is factive, "it seems" not.]
- There are inconsistent observations in the weak sense that one observation A may lead to accept thesis B, while another observation C leads to a rejection of thesis B. Oberservations (A and C) are then inconsistent with each other. This is an unproblematic everyday case.
- There are optical illusions, think of a picture by Escher. One seems to see an inconsistent objects. Of course, since these are illusions, there *is* no such object, but the *seeming* nevertheless is real. That seeming is, however, not stable (as you recognise if you try to fix both the contradictory perspectives on such a drawing in question).
- As one can observe A, one may observe $\neg$A (by noting the absence of A). As one can observe A and B, one may observe A $\wedge$ B. Now if observation is thus compositional, one may observe A $\wedge$ $\neg$A, as Priest (2002) claims. A $\wedge$ $\neg$A being *observable* (a modal claim) can only mean that there are situations/worlds where A $\wedge$ $\neg$A really is *observed*.

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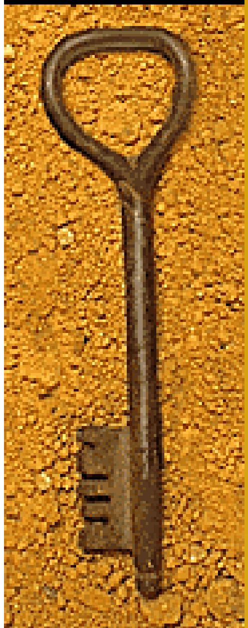
Sources of Inconsistency

- One can try to give a systematization how inconsistency *can arise* within a scientific theory (cf. Miller 2002):
- (a) reasoning from incorrect, or unknowingly restrictive, experimental data.
  This leads to inconsistency when the incorrect data are confronted with correct data, or when the overgeneralisation founded in the too restrictive data is put to the general test.
  [As examples are given Galileo with several of his experiments and the famous Michelson/Morley experiment!]
- (b) reasoning from incorrectly interpreted premises.
  [As example the supposed inconsistency of the wave/particle duality is given, taking "wave" and "particle" as *ordinarily* understood.]
- (c) reasoning on the basis of concepts that are later jettisoned.
  This by way of equation or by implicitly reasoning with principles (valid for the jettisoned concepts) the negations of which are in force in the updated theory. [An example may be Lorentz' ether theory.]



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Theory Revision

- Another application of weak paraconsistency can be theory revision.
- Theory revision or updating is standardly modelled by operations like theory expansion (throwing in more sentences), or contraction (throwing out some sentences).
  The standard methodology of the AGM-approach aims at preserving or regaining *consistency*. [see (Gärdenfors 1988)]
- A paraconsistent logic could claim to be *the* logic of theory revision in that it can accomodate inconsistent belief sets and their evolution into a new theory.
- The central concept of revision and the occasion to give up some belief by contraction can then no longer be consistency, but some other quality of belief systems. *Coherence* may be a candidate. Consistency no longer would be the first requirement on coherence, but other criteria of coherence like explanatory power, simplicity or data adequacy would not be modified. Even an inconsistent belief system might aim at coherence in that respect – as far as possible.
  [On coherence and its criteria see: (Bender 1989), (BonJour 1985), (Rescher 1979), (Thagard 1978).]

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Theory Revision (II)

- Formally the standard methodology of the AGM-approach is devleoped by axioms for expansion, contraction and revision (as comprised of the two former operations).
- If inconsistent belief are allowed the underlying logic has to be changed to a paraconsistent one, since trivial belief systems still don't make sense.
- Those axioms that lay down consistency [axiom (K5) for revision in (Gärdenfors 1988), basically] have to be given up.
- Revision has to be defined in a new way along the lines of the so-called "Levi Identity":

  Revision is expanding with A a contraction with ¬A.
- Revision in this sense is paraconsistently meaningful and may increase coherence (another AGM-axiom, (K7), has to be given up, however).

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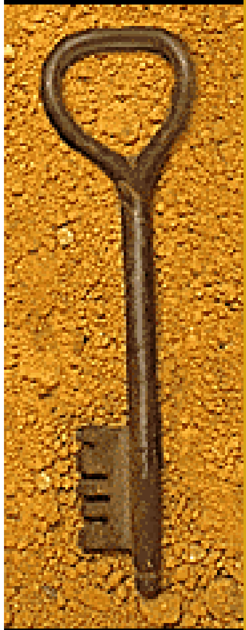
Coherence and Universality

- Epistemology itself may be considered to be at least implicitly inconsistent, since it is universal (employing semantic concepts).
- Epistemology is universal in that it talks *in general* and *universally* about the conditions of being true or being justified, including the justifiability or truth of epistemological theories. (It has to be universal given its philosophical aims and the implausibility of a hierarchy.)
- Universality is especially obvious in case of coherence theories. Coherence theories claim that given an alternative of theories we shall chose the theory that is more *coherent* than the other – whatever that may mean in detail.
- The coherence comparison is necessary, since more than one theory can cope with the data given, so that there are no "external" points of comparison if that meant confronting theories with the world. Rather theories have also to be compared with each other in the light of the coherence criteria (like explanatory power, completeness...).

Coherence and Universality (II)



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- Now, if they are comparable – and arguments against inconsummerability show that they are – there has to be a *super-frame* in which we do the comparison, or an *area of overlap of methodology*.
- This area of overlap or the super-frame consists of universal concepts of (theory) appraisal like *justified, explanatory, contained in* etc., concepts that also include the structural concepts of language.
- That we need this super-frame has to be justified within a theory of coherence (starting from a rejection of both naive realism and relativism), and it is another way – besides insisting on semantic closure [cf. Chap. 2] – to justify and outline the proper place of philosophy (as the general reflection on our (linguistic) ways of accessing reality).
- The standpoint of the super-frame or the standpoint of comparing complete webs of beliefs are universal in character and thus – at least for their containing semantics – are beset with antinomies.
- Coherence needs paraconsistency, even if consistency is a *criterion* of coherence for some lower level theories.

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Epistemic Logic

- Standard Epistemic Logic is a version of modal logic.
- As such it has been controversial in its closure assumptions like

  (1) $\vdash A \Rightarrow\ \vdash KA$

  (2) $\vdash (A \supset B) \Rightarrow\ \vdash (KA \supset KB)$

  (3) $\vdash KA \Rightarrow\ \vdash KKA$

  where "K" is the knowledge operator. (1) claims complete knowledge of logic, (2) closure of our knowledge under valid implications, and (3) the self-accessibility of our knowledge. All these claims have been criticised.
- If one tries to make it applicable to inconsistent belief systems one has to drop the axiom schemes requiring consistency of belief:

  (4) $\neg\text{BELIEF}(A \wedge \neg A)$

  (5) $\text{BELIEF}(\neg A) \supset \neg\text{BELIEF}(A)$

  Somebody may believe either an explicit contradiction – vs. (4) – or she may have an inconsistent set of beliefs, such that (5) is not true.
- Giving up (1) – (5) for reasons of psychological adequacy, however, does not leave much of a logic worth its name!

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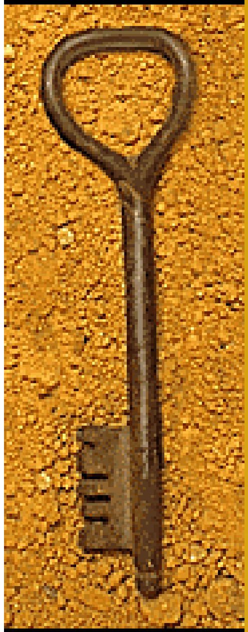
Inconsistent and Impossible Worlds

- Although "inconsistent world" and "impossible world" are often used interchangeably the two concepts should be kept apart.
- An *inconsistent world* is a world which for some sentence A contains both A and ¬A. The world (or the set of sentences it supports) is closed under the consequence relation of the underlying/assumed logic. An inconsistent world thus contains inconsistent information or inconsistent assumptions but does stick to the underlying/assumed logic. In case that logic is non-paraconsistent the world, of course, is trivial. If inconsistent worlds are to be non-trivial the underlying logic has to be a paraconsistent logic.
  [Note: We do not care in this chapter what worlds *are* or whether there are inconsistent objects; see Chap. 17. Worlds may be assumed here to be just maximal sets of sentences, i.e. they contain at least A or ¬A for any sentence A of the language used.]
- An *impossible world* is a world where the laws of the underlying logic need not hold, e.g. by non-compositional evaluation of complex sentences $A \supset B$ or $A \rightarrow B$. Impossible worlds are so used in Relevant semantics [cf. Chap.5] to invalidate *verum ex qoudlibet sequitur*.



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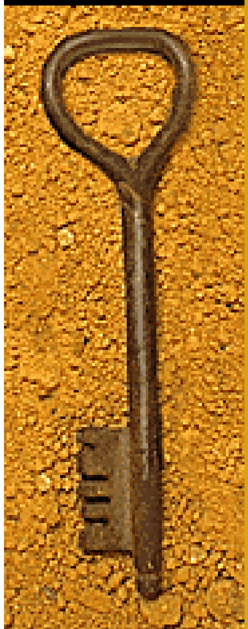
Impossible Worlds

- Whether impossible worlds have besides their role in Relevant modal semantics further philosophical functions is controversial.
- Some argue that we use impossible worlds when we reason and argue about which logic or set theory may be the correct one. But that seems rather to be a case where we – using some supposedly uncontested minimal logic – compare and argue whether a set of sentences should be closed under this or that consequence relation assuming these or those axioms. Considering what *is true according to some theory/logic* assumes *for the sake of the argument* that the theory/logic is adequate and reasons according to this. This is not the "anything goes" of impossible worlds.



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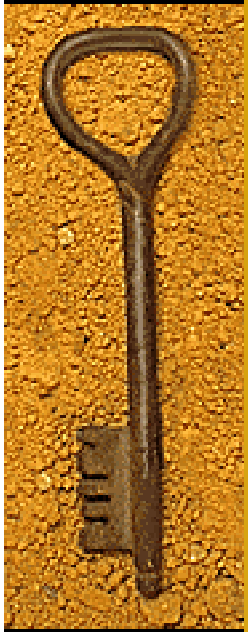
Impossible Worlds (II)

- Especially, impossible worlds are no counter-instances to a supposed *universal logic* [cf. Chap. 20]. Having a universal logic means to claim possessing a logical framework that can deal not just with the topics of common talk and the sciences, but also with the paradox ridden fields of semantics or set theory. Universal logic does not invite an "anything goes" or claims to deal with it. Universal logic pertains to how our logical faculties, supposedly, *really are*, not to collection of arbitrary evaluations that impossible worlds are. By arbitrary evaluation the supposed ordinary meaning of terms and compositionality are given up for grabs by *fiat*. One should not conclude that there aren't such meanings from one's own decision to ignore them (in evaluations).



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Impossible Worlds in Epistemic Logic

- The problem of omniscience in epistemic logic can be dealt with by employing impossible worlds.
- The problem was that our beliefs are not closed under logical consequence whereas epistemic modal logic claims just that.
- An epistemic logic with impossible worlds, where the usual compositional evaluation conditions do not apply, may have a restricted version of epistemic necessitation, restricted to the logical laws known by the agent in question (i.e. holding in all the worlds, including maybe impossible worlds, which are compatible with the agents beliefs).
- Let Ω be the set of known or believed logical laws, and let **PC** be the logic to be extended, then an omniscience restricting epistemic logic is:

  (PC) All PC-theorems are theorems.

  (K1) $KA \supset A$

  (K2) $KA \wedge K(A \supset B) \supset KB$

  (MP) $A,\ A \supset B \Rightarrow B$

  (NK) $A,\ A \in \Omega \Rightarrow KA$

Impossible Worlds in Epistemic Logic (II)

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- So, by (K2) an agent detaches the logical consequence of what she knows (or believes), but the logical consequences she takes into account depend, by (NK), on whether the implication in question is included in her logic knowledge set Ω.
- The semantics of this logic requires that even if these sentences may also be false in the impossible worlds the *known* logical laws are also true in the belief worlds of that agent.
- Truth in a model is defined – as in similar Relevant semantics – as truth in all *normal* worlds.
- The logic can be shown to be adequate given this semantics.

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Explicit vs. Implicit Belief

- Another way to avoid the counter-intuitive consequence of closure of belief (especially Explosion) is to distinguish explicit belief and implicit belief.
- Whereas one may claim that one is implicitly committed to all the consequences of what one believes one need not *explicitly* draw these consequences.
- Distinguishing an operator for *implicit* beliefs from an operator for *explicit* beliefs Levesque (1984) introduces a new epistemic logic.
- Explicit belief obeys closure under the axioms for entailment in this logic! That is, one only believes Relevant consequences of one's explicit beliefs explicitly. Neither closure nor Explosion hold for explicit belief.
- The standard closure principles (including Explosion), however, hold for implicit belief. – That again seems problematic, since any contradictions thus forces one to implicitly believe anything. Even so this is implicit belief, it is just too much.

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Rejoinder on Closure in Epistemic Logic

- One hypothesis on propositional attitudes might be that the very point of propositional attitudes is to block closure (i.e. failure of closure is no failure at all). Why should closure be blocked? Because – so one might argue – people may believe/intend… contradictory propositions/sentences/states… *and* our otherwise employed (real) logic is **PC**! So to avoid Explosion in reporting someone's believes or in stating one's owns beliefs closure has to be block (e.g. by a failure of adjunction of beliefs). So – the argument might continue – the failure of closure is only a problem for paraconsistent logicians and their claim that there is no usage of *ex contradictione* in the first place!
- Now, although the hypothesis may sound interesting at the beginning, our reluctance to let our inconsistent beliefs and theories explode (and become trivial) sheds doubt on the claim that we use the full force of **PC**, as argued before. Further on, if, as some theories of cognitive representation claim, all we believe/assert… stands – implicitly – in the scope of a "I believe/assert… that"-operator, then we employ closure principles like adjunction and detachment all the time, at least in our own case. The hypothesis has more force in the case of reporting someone else's beliefs which we know to be inconsistent. So there may be restricted closure in 3rd person reports on beliefs…

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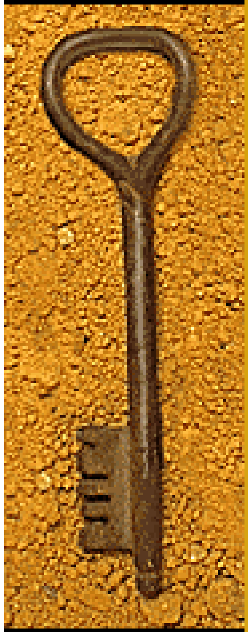
Inconsistent Worlds and Fiction

- Some works of fiction are inconsistent, but as with inconsistent theories that does not mean that they are trivial (i.e. claim that everything is the case in the world described). So with respect to such fiction a paraconsistent logic is needed.
- These works of fiction, on the other hand, *rely* on the reader drawing conclusions (i.e. they employ and stick to some logic). Because stories rely on the reader to fill in all the missing details (like persons mentioned having a birthday, the sky being above the ground …) they *require* that large parts of our common background knowledge and our logic are still in force. An impossible world, by contrast, needs to be specified completely in advance, since all or at least too many sentences are logically independent of each other.



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Paradoxes of Confirmation

- Although this is not directly related to paraconsistency, Relevant Logics are able avoid some of the paradoxes of confirmation theory.
- One standard is the *Raven Paradox*. The hypothesis

  (1) All Ravens are black./$(\forall x)(\text{Raven}(x) \supset \text{Black}(x))$

  can be confirmed, supposedly by positive instances. If a sentence is confirmed, then so should be all the sentences that are logical equivalent to it; and these are in FOL, for example,

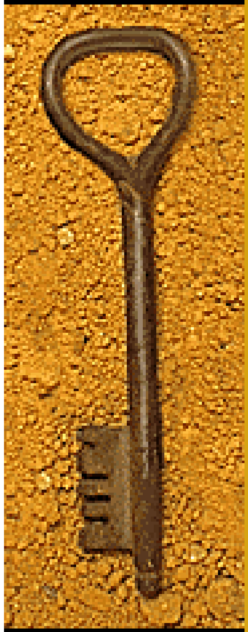
  (2) $(\forall x)(\text{Raven}(x) \wedge \neg\, \text{Black}(x) \supset \text{Raven}(x) \wedge \neg\, \text{Raven}(x))$

  (3) $(\forall x)(\neg\, \text{Black}(x) \supset \neg\, \text{Raven}(x))$

  Now, (2) it seems can never be confirmed, since its consequence contains a contradiction; (3) has the – supposedly paradoxical – consequence that every instance of a non-black thing (e.g. a piece of chalk) supports a thesis about ravens.
- In many PLs (e.g. in **SKP**) neither (2) nor (3) is logical equivalent to (1), so even maintaining the thesis of confirmation equivalence of logical equivalents the *Raven Paradox* does not ensue.

Assessment

- Within the philosophy of science we can see weak paraconsistency at work. There is plenty evidence that we need some weak paraconsistent modelling in some examples from the history of science and in theory change.
- There is no need for dialetheism in the philosophy of science, since scientists, although working for some time with inconsistent theories, still assume that reality is consistent and thus the ultimate theory will be consistent.
- In case of epistemology the general argument tying universality in philosophy to dialetheism applies. A universalist – "transcendental" – philosophy of knowledge can only be dialetheist.



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Questions

- (Q1) Why does the Raven Paradox not arise in **SKP**? That is: why can't (2) or (3) not be derived from (1)?
- (Q2) $A \wedge \neg A$ being *observable* (a modal claim) can only mean that there are situations/worlds where $A \wedge \neg A$ really is *observed*. It is not just a seeming! How could that be? – Try it for yourself!
- (Q3) Within philosophy of science there has been a shift from a model of theories as logically closed sets of sentences towards the so called *structuralist* theory (Sneed 1979, Stegmüller 1986), which sees theories as defined by models in the manner of set theory. The structualist picture distinguishes between the core of a theory and its full models and intended applications. Is this picture less opposed to the idea of inconsistent theories than the standard view (so called "statement view") of theories?

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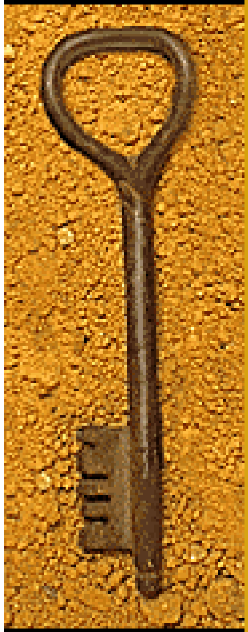
Exercises

- (Ex1) In case you are familiar with the AGM-approach to belief revision, try to find a paraconsistently possible case of belief revision in which axiom (K7) for revision is invalidated.
- (Ex2) Go through a model case of logical omniscience being restricted to some Ω, using (K1), (K2), (NK).



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Further Reading

- A reader dedicated solely to paraconsistent modelling and interpretation of inconsistency in science is: Meheus, Joke (Ed.) *Inconsistency in Science*. Dordrecht, 2002.

  The examples in this chapter are mostly taken from that book. The essays in it deal both with static as well as dynamic inconsistency (using adaptive logics).
  In that book you also find (Priest 2002) and (Miller 2002).
- On paraconsistent belief revision cf. Fuhrmann, Andre. "Theory Contraction Through Base Contraction", *Journal of Philosophical Logic*, 20 (1991), and especially: Tanaka, K. "What Does Paraconsistency Do? The Case of Belief Revision", *The Logica Yearbook 1997*, ed. by T. Childers, Prague, 1998, pp. 188–197.
- On standard Epistemic Logic see (Lenzen 1980). A critic of its closure conditions you find in (Hintikka 1970 and 1973). On its paraconsistent restriction see (Routley 1979a, pp. 884-94).

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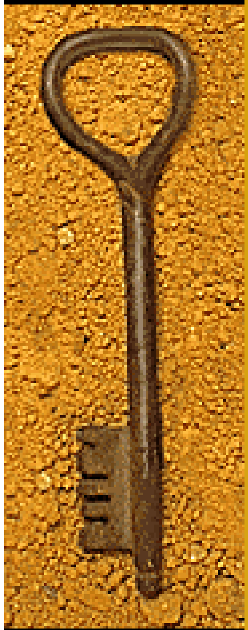

Further Reading (II)

- The *Notre Dame Journal of Formal Logic* devoted an issue, Fall 1997, to theories of impossible and inconsistent worlds, edited by Graham Priest.
- Veikko Rantala applied impossible worlds to the omniscience problem of epistemic logic; cf. "Impossible Worlds Semantics and Logical Omniscience", *Acta Philosophica Fennica*, 35 (1982), pp.106-15.

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