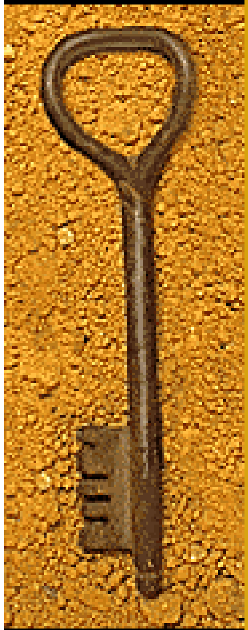
A Bunch of other Systems



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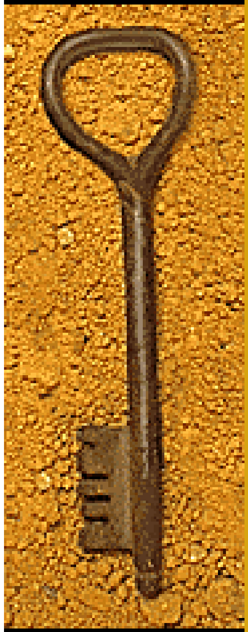
- The minimal condition to call a logical system "paraconsistent" is that it avoids Explosion or *ex contradictione qoudlibet*.
- There are many ways to do this. Dialetheism and even weak paraconsistency tie the logical system investigated to more or the less specific programes in philosophy. There may be other philosophical programs that look out for a suitable paraconsistent system.
- One may, further on, completely drop the link to some philosophical program or thesis and investigate a manifold of logical systems for its own sake, as an excercise in the pure investigation of logical structures that may widen the horizon restricted by a narrow focus on standard logics.
- In this chapter a couple of systems are presented that either do not fit into one of the programs or approaches presented so far, or are given as mere logical systems, which simply are paraconsistent.



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RM3

- **RM3** is a system that can be characterized truth-functionally (like **LP**), but that in contrast to **LP** keeps *Modus Ponens* for " $\supset$ ". As a consequence of this **RM3** cannot – as does **LP** – keep all **PC** tautologies, since otherwise Explosion would be valid.
- **RM3** has some features of a Relevant Logic. It is Relevant in having neither *ex contradictione qoudlibet* nor *verum ex qoudlibet sequitur* neither as theorems nor as rules.
- Because *verum ex qoudlibet sequitur* does not hold, but **RM3**-consequence is monotonic (i.e. $A \vdash B \Rightarrow A,C \vdash B$) **RM3** cannot have the *Deduction Theorem*!
- Designated truth values in **RM3** are 1 and (0,1).

- **RM3** can be characterized by the following truth tables:

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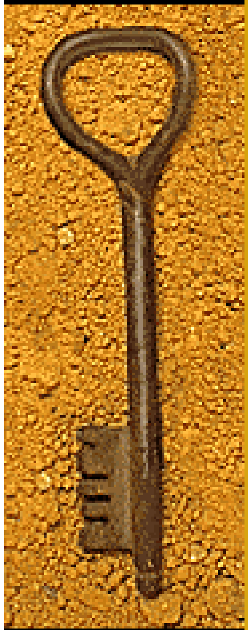
RM3 Truth Tables

A	B	¬A	A∧B	A∨B	A⊃B	A≡B
1	1	0	1	1	1	1
1	0	0	0	1	0	0
1	0,1	0	0,1	1	0	0
0	1	1	0	1	1	0
0	0	1	0	0	1	1
0	0,1	1	0	0,1	1	0
0,1	1	0,1	0,1	1	1	0
0,1	0	0,1	0	0,1	0	0
0,1	0,1	0,1	0,1	0,1	0,1	0,1



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RM3 Theorems

- **RM3** has not only *Modus Ponens*, but also Contraposition and so *Modus Tollens*.
- The conditional cannot be defined in the usual way. We have neither

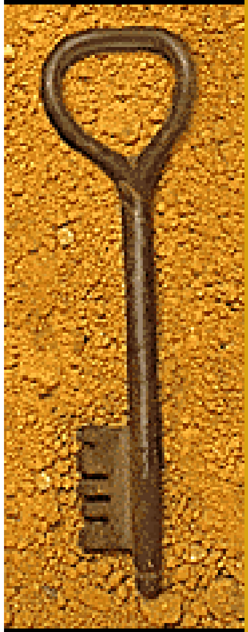
  $$(A \supset B) \equiv \neg (A \wedge \neg B)$$

  nor

  $$(A \supset B) \equiv \neg A \vee B$$
- **RM3** truth tables differ from **LP** truth tables in the evaluation of " ⊃ ". In case of a 1 antecedent and a (0,1) consequent the conditional is just false in **RM3** whereas it is true and false in **LP.** The **LP**-evaluation looks more natural, since in the 1:1-reading the conditional is true, and in the 1:0-reading it is false. The same holds for a conditional with a (0,1) antecedent and a 0 consequent, which is only false in **RM3**.
- The biconditional, because of this, at first is more natural than in **LP**, since it behaves binary in case of the standard values. In case of a (0,1) right hand and left hand side, however, it only gets the value (0,1).

RM3 Theorems (II)

- **RM3** has the *Modus Ponens*-Theorem, so it falls short of the Strong Anti-Triviality Condition [cf. Chap. 3].



p ∧¬p

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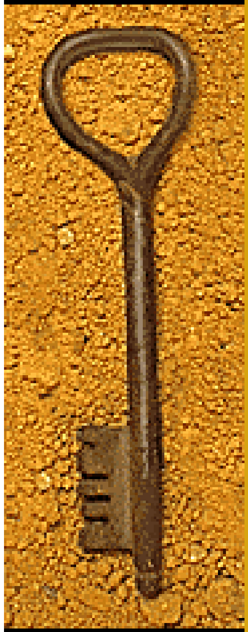
BN4

- So far we have not concerned logics which allow for truth value gaps. The existence of gaps is a question that can be kept apart from considerations of paraconsistency. There may be applications of paraconsistency, however, in which the existence of gaps may be useful [if one deals with vagueness or failure of reference in this way (e.g. Blau 1978), see also Chap. 14 on Deontic Logic].
- **BN4** extends **RM3** by taking also truth value gaps into account (so its theorems are a subset of the **RM3** theorems).
  [Note that everything we said about **RM3** theorems and evaluations also applies to **BN4**.]
- The fact that some sentence A has no truth value means that the evaluation function (which assigns subsets of the set {0,1} of truth values to each sentence) assigns ∅ to A.
- In **BN4** – in distinction to **LP** – the Deduction Theorem does *not* hold.
- **BN4** can be given by the following truth tables:



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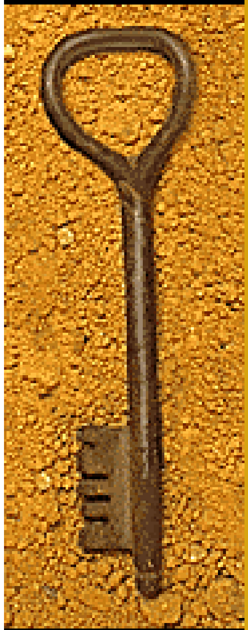
BN4 Truth Tables

A	B	¬A	A∧B	A∨B	A⊃B	A≡B
1	1	0	1	1	1	1
1	0	0	0	1	0	0
1	0,1	0	0,1	1	0	0
1	∅	0	∅	1	∅	∅
0	1	1	0	1	1	0
0	0	1	0	0	1	1
0	0,1	1	0	0,1	1	0
0	∅	1	0	∅	1	∅
0,1	1	0,1	0,1	1	1	0
0,1	0	0,1	0	0,1	0	0
0,1	0,1	0,1	0,1	0,1	0,1	0,1
0,1	∅	0,1	0	1	∅	∅
∅	1	∅	∅	1	1	∅
∅	0	∅	0	∅	∅	∅
∅	0,1	∅	0	1	∅	∅
∅	∅	∅	∅	∅	1	1



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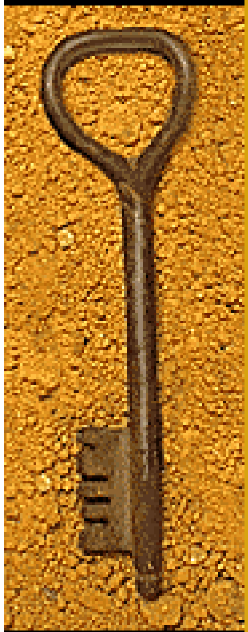
Four

- **BN4** was considered by Belnap to be the appropriate logic of logic programming and artificial intelligence.
- Damasio and Pereira (1998) modify **BN4** in changing the truth table for the conditional connective making it bivalent, and defining

  $$\models_{FOUR} A \Leftrightarrow v(A)=\{1\} \text{ or } v(A)=\{0,1\}$$

  with 1 and (0,1) being the designated truth values.
  They call the resulting logic **FOUR**.
- The associative, distributive, commutative, identity, double negation and DeMorgan laws all hold in **FOUR**. So does Contraction. *Modus Ponens* holds. The Deduction Theorem applies.
- The duals for the conditional do not hold. Neither does *Tertium Non Datur* nor the *Law of Non-Contradiction*. *Modus Tollens* and Disjunctive Syllogism fail.



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FOUR Truth Tables

A	B	¬A	A∧B	A∨B	A⊃B	A≡B
1	1	0	1	1	1	1
1	0	0	0	1	0	0
1	0,1	0	0,1	1	1	0
1	∅	0	∅	1	0	0
0	1	1	0	1	1	0
0	0	1	0	0	1	1
0	0,1	1	0	0,1	1	0
0	∅	1	0	∅	1	1
0,1	1	0,1	0,1	1	1	0
0,1	0	0,1	0	0,1	0	0
0,1	0,1	0,1	0,1	0,1	1	1
0,1	∅	0,1	0	1	0	0
∅	1	∅	∅	1	1	1
∅	0	∅	0	∅	1	1
∅	0,1	∅	0	1	1	1
∅	∅	∅	∅	∅	1	1

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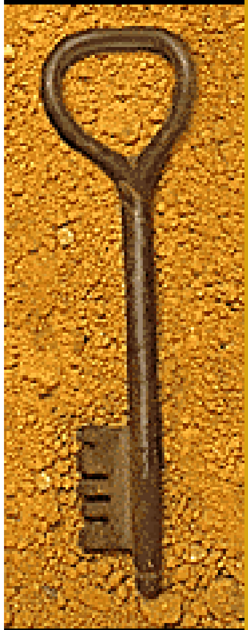
Dialogical Paraconsistent Logics

- Dialogical Logics are logics in the tradition of Kuno Lorenz and Paul Lorenzen (Lorenzen/Lorenz 1978).
- In dialogical logics the rules of the logical particles (connectives as well as quantifiers or modal operators) are introduced as moves of *attack* and *defense* between two players. The proponent proposes a theorem which is then attacked by an opponent.
- Frame rules regulate how often an attack can be repeated (e.g. on both sides of a conjunction or on a universal quantifier) and which attack can be chosen to defend against.
- One may see dialogical logics as a pedagogical device to introduce the operation of logical rules. (As in everyday argumentation this might be the way we challange somebody's claims.)
- Interessentingly the choice of frame rules determines whether one gets standard or – in case the proponent may only defend against the last attack made – intuitionistic logic!



Dialogical Paraconsistent Logics (II)

- Rahman and Carnielli have introduced a paraconsistent dialogical logic by further modifying the frame rules.
- The paraconsistent constraint says that a proponent can attack only those negated sentences of the opponent (by claiming the unnegated sentence as true) that the opponent has attacked in the same dialogue himself (i.e. with respect to which the opponent has claimed the unnegated sentence himself).
- This further constraints leads to the invalidity of Explosion and several versions of Disjunctive Syllogism.
- No longer valid – in comparison to intuitionistic logic which is the base for the more restrictive frame rules – are also dualities for the conditional connective. DeMorgan's Laws – even the intuitionistically acceptable direction – however also fail.

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Different Names

- Some paraconsistent logics that were around before we have dealt with in their LFI-systematization: **Pac** and **J3** are subsystems or extensions of **LFI1**.
  [There's a chapter on **J3** in (Epstein 1995).]

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More Systems

- If you don't care about the criteria given in chapter 3, you can have the queerest system you like, violating more than one of the criteria, but being paraconsistent by invalidating Explosion.
- (Popov 1988) introduces some other paraconsistent sequential calculi than **SKP**. The semantics seem highly contra-intuitive and leads to the lack of such extensional rules as Conjunction Elimination. Similar calculi you find in (Scotch/Jennings 1989), where also *Modus Ponens* fails!
- Nelson (1949, 1959) developed a version of intuitionism in which appear sentences which are true and false at the same time. To have this Nelson introduces a further – even more "constructive" – negation. In some versions of his systems neither Absorption nor Contraposition hold. The systems are proof theoretically adequate relative to a intuitionistic Kripke semantics.
- (Arruda/da Costa 1984) and (Routley/Loparic 1980) discuss versions of Relevant logics with quite strange five valued truth tables.

Questions

- (Q1) **BN4** truth tables including assignments of ∅ look highly arbitrary: Why is a conditional of indeterminate sentences true? Why is a disjunction between an antinomic sentence and an indeterminate sentence simply true? Can you find any *rationale* for this?
- (Q2) What can be said about **FOUR** in the light of our criteria for paraconsistent logics?
- (Q3) Why does – given **RM3**-semantics – monotonicity of $\vdash$ block the *Deduction Theorem*?

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Exercises

- (Ex1) Show by **RM3** truth tables that neither

  $$(A \supset B) \equiv \neg (A \wedge \neg B)$$

  nor

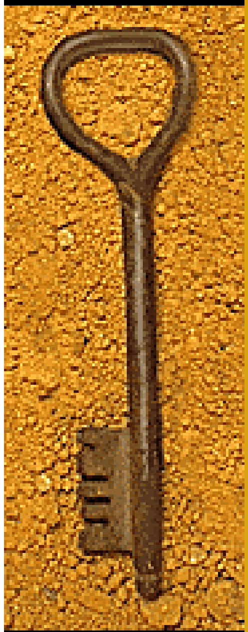
  $$(A \supset B) \equiv \neg A \vee B$$

  are valid in **RM3**.
- (Ex2) Show by **BN4** truth tables that Contraposition, Double Negation and DeMorgan's Laws hold.
- (Ex3) Show by **BN4** truth tables that Contraction does not hold.
- (Ex4) Show that Negation Introduction fails in **FOUR**:

  $$(A \supset B) \supset ((A \supset \neg B) \supset \neg A)$$

Further Reading

- On the systems **RM3** and **BN4** see: Brady, Ross. "Completeness proofs for the systems **RM3** and **BN4**", *Logique et Analyse*, 25, pp. 9-32.
- On paraconsistent dialogical logic see: Rahman, Shahid/Carnielli, Walter. "The Dialogical Approach to Paraconsistency". Fachrichtung Philosophie der Universität des Saarlandes, Report No. 8. Saarbrücken, 1998, 33pp.; Rahman has worked on several extensions of dialogical logic, including inconsistent ontology and multi-modal systems; you can find there also an introduction – in English – to dialogical logics in general.



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