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Methodological Considerations

- The paraconsistent approach being justified in general we have to consider in which direction our reshaping of logic should proceed.
- It won't be the case that any direction will do – in short, we need:
  a) *goals* that the logic we look for should satisfy,
  b) *criteria* that the logic has to meet on its way.
- The goals in (a) have prominently to include:
  - avoiding Explosion (i.e. invalidating at least some forms of *ex contradictione qoudlibet*)
  - being able to model semantic closure [given the discussion in Chap. 2]
- The criteria in (b) may contain features of a logical system that we are not ready to give up even when going paraconsistent. Any logical system not meeting these (necessary) conditions should be disregarded. If there were no paraconsistent systems satisfying these conditions, we would have either to reconsider the criteria – but this would be a desperate measure pointing to the insufficiency of paraconsistent logics – or we would have to turn to some alternative to paraconsistency if there is any.

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Irrelevant Implications

- *ex contradictione qoudlibet* in its several versions

  (1) $A \supset (\neg A \supset B)$

  (2) $A \wedge \neg A \supset B$

  is one of the so-called "paradoxes of material implication": sentences that are logically true, although they are highly contra-intuitive. Other examples are:

  (3) $A \supset (B \supset A)$

  (4) $A \supset (\neg B \vee B)$

- (3) and (4) are versions of *verum ex qoudlibet sequitur*. The truth (i.e. any true sentence) is said to follow from everything (since whatever the antecedents may be, the consequence is true).
- In all these examples there is no connection between the content of the antecedent and the content of the consequent. Our intuitive notion of a conditional, on the other hand, is biased in favour of some causal or semantic connection between the parts of a true conditional. Relevant Logics (RLs) try to model a conditional connective along these lines.
- For a paraconsistent logic this could be a goal as well. At least Explosion as a rule $[A, \neg A \vdash B]$ has to go.

Blocking Explosion

- *ex contradictione qoudlibet* [A, $\neg$A $\vdash$ B] has to go; that means that something involved in its derivation in PC has to go:
1.<1> p	PREM
2.<2> $\neg$p	PREM
3.<1> p $\vee$ q	($\vee$I), 1
4.<1,2> q	($\vee$E), 2, 3
5.<1> $\neg$p $\supset$ q	($\supset$E), $\underline{2}$, 4
6.<> p $\supset$ ($\neg$p $\supset$ q)	($\supset$E), $\underline{1}$, 5 ■
- The last two steps derive Explosion as a theorem by conditionalization, even without them we would have Explosion as a deductive rule.
- Looking at the derivation, we see that there are not many options to choose from: We have to drop
  - -*either* ($\vee$I)
  - -*or* ($\vee$E)
  - -*or* we could modify some meta-logical properties of the relation of provability, namely its transitivity, such that A $\supset$A$\vee$B and (given the standard definition of "$\supset$") A$\vee$B $\supset$($\neg$A $\supset$ B) hold, but A $\supset$ ($\neg$A $\supset$ B) does not!

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Blocking Explosion (II)

- Giving up the transitivity of provability seems to be a desperate measure and an unwanted result if it was to hold of some PLs, $\vdash$. Transitivity seems to be both a fundamental ingredient of our understanding of an implication and of joining two parts of a proof. Keeping the transitivity of the conditional connective or at least the transitivity of provable implication is a criterion [in the sense of (b) above] for our target logics. So I won't consider the third alternative here, although at least one author has in fact taken that route (cf. *Further Reading*).

- Note: The problem of Irrelevance, that antecedent and consequent share no content, cannot simply be solved by requiring that they have to share some formula (in the propositional case).
Since $B \wedge \neg B \supset B$ certainly holds [by ($\wedge$E)] and fulfils the requirement, we have $A \wedge \neg A \supset B$ once we admit that all contradictions are equivalent, which is *logically true*, $\vdash (A \wedge \neg A \equiv B \wedge \neg B)$, and allow for the substitution of proven equivalents! Giving up the substitutivity of proven equivalents seems to be another severe revision of our intuitive understanding of logical matters, but may be unavoidable.

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Or

- Concerning the contra-intuitiveness of giving up transitivity the two other options do not seem to fare better. Consider the standard truth-table of disjunction:

A	B	A∨B
T	T	T
T	F	T
F	T	T
F	F	F

- Given the truth of one disjunct it certainly holds that the whole disjunction is true [that corresponds to (∨I)] and given the falsity of one disjunct the truth of the disjunction has to depend on the truth of the other disjunct [that corresponds to (∨E)].
- Giving up one of these rules, therefore, seems to be revisionistic with respect to the meaning of "or", one of our most fundamental connectives! And that we should not allow too easily.

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Or (II)

- Giving up ($\vee$I) seems impossible. Once we have the truth of a sentence it won‘t go away if that very sentence becomes part of a disjunction.
- And there are only a few logics that try to do this.

[One, **ANA**, is discussed in chapter 7, but we will see that it has a very deviant semantics for "$\vee$"; Weingartner‘s approach (cf. Weingartner 1999) is not a logic, but a filtering of given derivability relations, and so is no help in modelling actual reasoning (i.e the derivation of the to be filtered set).
Another logic that invalidates ($\vee$I) is Bochvar's 3-valued logic (Bochvar 1939): The third value is taken as "meaningless" and this feature is inherited from a part of a complex sentence to the complex sentence. Thus Bochvar arrives at the so called Weak Kleene Tables for a 3-valued logic, in which the value "meaningless" is infectious. One can have A but not A $\vee$ B, since if B is meaningless, the whole compound is meaningless, and meaninglessness is, of course, no designated truth value. Bochvar's logic, however, is not even weakly paraconsistent, since it validates Explosion!]

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Or (III)

- Giving up ($\vee$E) is, therefore, the route taken in many paraconsistent logics.
- Here we can really think of a counterexample *without* having to modify the semantics of "$\vee$":
  If A is a true contradiction, then by it being true it makes A$\vee$B true, but we shouldn‘t employ Disjunctive Syllogism, because by A‘s being false, we could derive any B, even a B that is false, so we had a rule with true premises and a false conclusion!
- In fact even though A is false, and so, hopefully, $\neg$A true, that does not fit our description of reasoning from the falsity of one disjunct to the truth of the other: Since A is a dialetheia, A is not false simpliciter, and so that *rationale* does not apply!

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$\vee$ and $\supset$

- There is another problem if we give up Disjunctive Syllogism: In standard semantics the conditional connective is defined by
  (5) $p \supset q \equiv \neg p \vee q$
  or at least this holds as a theorem of duality.
- If that is so then giving up Disjunctive Syllogism forces one to give up
  $A \supset B, A \vdash B$ [being $\neg A \vee B, \neg\neg A \vdash B$]
  that is, ($\supset$E) or *Modus Ponens*!
- Detachment, *Modus Ponens*, however, is once again one of the most fundamental properties (if not *the* most fundamental property) we expect of a conditional connective. A connective not fulfilling *Modus Ponens* seems to be no *conditional* at all. How can it be a conditional if given the condition we are not allowed to reason to the consequent?
- Keeping *Modus Ponens* for the conditional connective is a criterion for any logic.
- For the PLs giving up ($\vee$E) this means that the semantics of the (main) conditional connective cannot be given by (5), but has to be worked out *independently* of "$\vee$". So their conditional is usually not the material conditional of **PC**.

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Negation

- Paraconsistency can also informally be defined by saying that it allows for some sentence and its negation to be true. Can the negation used such be our standard negation?
- This depends on what "standard" or "intuitive" negation is. If standard negation means negation in standard logic, then being standard contains validating Explosion and thus precludes paraconsistency. Negation, however, is a controversial issue in the debate about logics, the question being which conditions on a negation symbol in some logic capture our intuitive understanding of negation most adequately. If Explosion is intuitively unacceptable, then it better be not valid in a logic (at least a logic for such everyday matters).
- Two minimal conditions for negation in terms of semantics are:

  (MN) (i) If A is false, then the negation of A is true.

  (ii) If A is true, then the negation of A is false.
- This may be too much for someone who rejects *Tertium Non Datur*, but since we are concerned there mainly with paraconsistency we assume this to be not the focus of discussion

Negation (II)

- (MN) is easily fulfilled by standard logic, since in it "false" is the same as "not true" and *Tertium Non Datur* holds.
- (MN), however, may be equally satisfiable by non-standard negations and non-standard semantics, especially when "false" and "not true" come apart as in paraconsistency.
- So besides being extensional negation should satisfy (MN).
- Arguing for other *necessary conditions* for something being a negation often beg the question against paraconsistency by *defining* standard properties like Explosion into negation [e.g. (Slater 1995)]. The justification for conditions on negation should also not presuppose the absense of true contradictions or the identity of non-truths with falsity [(Lenzen 1998) for example argues for Contraposition in the form

  $$A \vdash B \Rightarrow \neg B \vdash (\neg A)$$

  to be a *necessary* condition for a "real" negation – thus excluding many paraconsistent negations –, but his justification of this definition presupposes that we reason on a meta-level with a standard negation not accepting contradictions or that B being false means B being not true in the sense of excluding B being true as well (cf. ibid, p.219), presuppositions not acceptable for a dialetheist.]

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There Has to Be a Logic

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- The most basic of all conditions for a PL is that we still have a logic. This means that we really have a system of derivation. Although this sounds trivial, there are two ways to deal with the antinomies that do not fulfill this criterion:
  a) an approach focussing on semantics only
  b) an approach focussing on the pragmatic usage of standard logic.
- Approach (b) although it might be interesting severes the link between our ordinary concept of argumentation and the process of deriving conclusions in some prove system. We usually expect logic to be a formalization of our ordinary ways of reasoning and arguing. And given that linkage it seems implausible that we use logic to derive some conclusions *and then* revise this set of conclusions to retract all Irrelevant deductions. A goal of formalization is that the steps taken in the logical system mirror in some systematic way our steps of reasoning. (In this manner logic proceeds preferably as a natural deduction system.) (b), therefore, seems to be a desperate measure. [Schurz (1991) goes that way by stating that reasoning is not to be identified with logic, but with "logic + relevance", relevance being modelled as a filtering of **FOL**-inference, i.e. being introduced *after deriving conclusions.*]

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Rescher/Brandom (I)

- The restriction to semantics [approach (a) above] we find in Rescher and Brandom's *The Logic of Inconsistency*:
- They assume *normal* possible worlds in which the status of A is independent of the status of $\neg$A, but $\neg A \wedge A$ is never the case. This means *being the case* of states of affairs is not truth-functional in "$\wedge$"! It does not hold: (1) $[A]_{w1}=1, [B]_{w1}=1 \Rightarrow [A \wedge B]_{w1}=1$
- There are, however, non-standard worlds (derivable as the union or the cut of normal worlds) in which $\neg A \wedge A$ occurs, and is true. Rescher and Brandom keep **PC** as logic, but they redefine the consequence relation: It no longer holds:

  (2) If $A_1 \ldots A_n \vdash B$ then $[A_1]_{w1}=1 \ldots [A_n]_{w1}=1 \Rightarrow [B]_{w1}=1$

  i.e. a syntactic conclusion of a premise set need not be true in a world although all the premises are true; instead of that we have only:

  (3) If $A_1 \ldots A_n \vdash B$ then $[A_1 \wedge \ldots A_n]_{w1}=1 \Rightarrow [B]_{w1}=1$.

  Now, in normal worlds (those worlds we may find ourselves in) the conjunction on the right hand side of (3) cannot be true if some A_j is the negation of A_i, so Explosion can be avoided.
- The non-standard worlds, however, contain, every sentence of the language. They are trivial!

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Rescher/Brandom (II)

- The non triviality of standard worlds is only kept, since (1) fails. That means that ($\wedge$I) fails!
- In their semantics even *Modus Ponens* is invalid. Take a world w_3 to be the union of the worlds w_1 and w_2 where we have:
  $[A]_{w1} = 1$, $[B]_{w1} = 0$, $[\neg A]_{w2} = 1$, $[B]_{w2} = 0$. In w_3 we have that A and $\neg$A are true, and so is $\neg A \vee B$ (i.e. $A \supset B$), but B is false, since B is false both in w_1 and w_2. So we have a counterexample to *Modus Ponens*.
- This approach, therefore, is unsatisfactory on several accounts:
  - the truth-functionality of the basic connective "$\wedge$" is given up
  - the conditional connective does not obey *Modus Ponens*
  - we have no method to deal with the possibility that our world is an inconsistent world; all sentences would be *derivable* then, although we could – somehow – turn to semantics to see which of these are "really" derivable. Why use logic then at all?
- [Furthermore – as Rescher and Brandom admit – the strengthened Liar can be introduced within their framework (cf. Rescher/Brandom 1980, p.34).]

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Criteria

- As we said we need some goals and criteria to assess paraconsistent logics:
- I. Avoiding Explosion and its relatives (otherwise the logic is not paraconsistent at all, unable to separate inconsistency from triviality). This we call the "Weak Anti-Triviality Condition".
- II. Any logic needs a conditional connective which has *Modus Ponens* as a rule. This we call the "Modus Ponens Condition".
- III. The other usual connectives (like negation, conjunction or disjunction) may not be redefined in a way which is at least as implausible as the paradoxes of material implication. This we call the "Extensionality Condition". Dualities like *DeMorgan* should hold true.
- IV. Rules of deduction we commonly associate with the conditional connective (or the extensional connectives in [III]) are not be given up lightly (for example the transitivity of that connective). This we call the "Minimal Damage Condition". It allows of degrees of meeting it.
- V. Naive Semantics and Naive Set Theory should be non-trivial given the logic in question. That is the main goal of dialetheism. This is called the "Strong Anti-Triviality Condition".

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Refining Criterion V. – Curry's Paradox

- The "strong anti-triviality condition" is philosophically justified by the arguments concerning semantic closure.
- We can refine this condition by explicitly stating which formula or rules should be avoided. This refinement is due to Curry's Paradox.
- Curry's Paradox comes in several versions.
  They show that given the validity of some formula Naive Semantics and Naive Set Theory are trivial.
- Let us consider two versions. Both use the usual conditional and bi-conditional, but the paradox also holds for strict implication.
  "( ) is true" is used as a predicate, but the paradox also holds if "True( )" is taken as a sentential operator.
- Define (A) as the sentence which states that if this sentence is true, then B is true:

  $$(A) = \text{True}(A) \supset B$$

  A semantically closed language has to contain a sentence like (A). By convention (T) we have

  $$(1)\ \text{True}(A) \equiv \text{True}(A) \supset B$$

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Curry's Paradox

- (1) $True(A) \equiv True(A) \supset B$

A logical truth ("Absorption"/"Contraction") for "$\supset$" is:

(2) $(A \supset (A \supset B)) \supset (A \supset B)$

Since (1) from left to right is an implication we get from (1) and (2):

(3) $True(A) \supset B$

Taking (1) from right to left this gives us:

(4) $True(A)$

and now with *Modus Ponens* we get with (3)

(5) B

where B is any sentence whatsoever! Since – given the *Modus Ponens* Condition – we won't give up *Modus Ponens*, we have to give up Absorption/Contraction as a valid formula (that is: to fulfill the "strong anti-triviality condition" the logic we choose has to invalidate Absorption/Contraction).

Curry's Paradox (II)

- The second version also starts from sentence (A) and convention (T):

  (1) $True(A) \equiv True(A) \supset B$

  We have as a logical truths:

  (2) $A \wedge (A \supset B) \supset B$

  as an instance we have

  (3) $True(A) \wedge (True(A) \supset B) \supset B$

  We can now substitute equivalents, given (1), and get:

  (4) $(True(A) \supset B) \wedge (True(A) \supset B) \supset B$

  by simplification of a conjunction we get:

  (5) $(True(A) \supset B) \supset B$

  Now the left side of (5) just is A and by (1) and transitivity we get:

  (6) $True(A) \supset B$

  and with (1) again from right to left we get: T

  (7) $True(A)$

  and with *Modus Ponens* (6), (7) finally:

  (8) B

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Curry‘s Paradox (III)

- In the second version we derive any sentence B whatsoever by using some seemingly innocent means of deduction:
  - transitivity of the conditional connective we won‘t give up (see the Minimal Damage Condition); the same holds for the substitution of provably equivalents (as the two formulas are given the definition);
  - *Modus Ponens* as a rule we won‘t give up easily because of the corresponding condition [see below];
  - simplification of conjunction we cannot give up without violating the Extensionality Condition of keeping ordinary conjunction.
- The only possible culprit here is, therefore, the *Modus Ponens Theorem*:

$$A \wedge (A \supset B) \supset B$$

- As with the first version of Curry’s Paradox this leads to a refinement of the "strong anti-triviality condition".

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Curry's Paradox (IV)

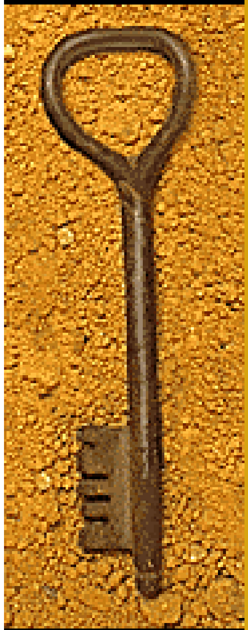
- Let us consider a 3rd version of Curry's Paradox which involves some reasoning about a proof system (as may occur in a sequential calculus) and applies *Modus Ponens, Conditionalization* ($\supset$Introduction), and *Cut/Transitivity of Consequence*:
  Semantic Closure should entail that True(A) and A are provably equivalent:

  $$\text{(TE) True(A)} \vdash \text{A and A} \vdash \text{True(A).}$$

  Now, given the Curry Sentence: (1) $\text{True(A)} \equiv \text{True(A)} \supset \text{B}$

  we may reason as follows:

1. $\text{True(A)} \vdash \text{True(A)} \supset \text{B}$	(TE) with (1)
2. $\text{True(A)}, \text{True(A)} \supset \text{B} \vdash \text{B}$	Instance of *Modus Ponens*
3. $\text{True(A)} \vdash \text{B}$	Cut 1, 2
4. $\vdash \text{True(A)} \supset \text{B}$	($\supset$I), 3
5. $\text{True(A)} \supset \text{B} \vdash \text{True(A)}$	(TE) with (1)
6. $\vdash \text{True(A)}$	Cut 4,5
7. $\vdash \text{B}$	Cut 3,6 ■



Curry's Paradox (V)

- *Cut/Transitivity of Consequence* we will not give up easily, since transitivity is one of the main properties a consequence relation is expected to have.
- So taking the intersubstitutivity of True(A) with A as beyond criticism and *Modus Ponens* as hardly to be given up the problem with this last version of Curry's Paradox is the usage of *Conditionalization*, a rule as hard to be doubted as *Modus Ponens*!

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Curry‘s Paradox (VI)

- Finally let us consider a 4th version which uses Identity

$$A \supset A$$

and the following version of transitivity:

$$(+) \qquad (A \supset (B \supset C)) \supset (A \supset B) \supset (A \supset C))$$

- Define the Curry Set $c = \{x \mid x \in x \supset D\}$

1. $c \in c \supset (c \in c \supset D)$ — Naive Comprehension and df. of c
2. $(c \in c \supset (c \in c \supset D) \supset ((c \in c \supset c \in c) \supset (c \in c \supset D))$ — (+) scheme
3. $((c \in c \supset c \in c) \supset (c \in c \supset D))$ — Modus Ponens 1,2
4. $c \in c \supset c \in c$ — Identity instance
5. $c \in c \supset D$ — Modus Ponens 3,4
6. $c \in c$ — df. of c with 5
7. D — Modus Ponens 5,6 ■

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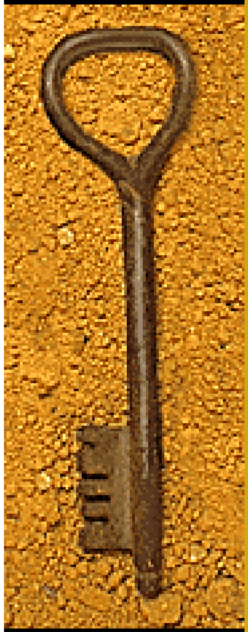
Curry's Paradox – No End in Sight?

- There might be more version of Curry's Paradox; but – fortunately – no more are known. At least none that use standard principles.

- Given some – slightly strange – conditional theorems, like
  $$((A \supset B) \supset B) \supset A,$$
  one may have more versions of such paradoxes
  [cf. (Rogerson/Restall 2004)].

- [Note: (Smiley 1993), for example, tries to give another serious version, but he uses rules like contraposition that need not be paraconsistently valid, and uses a principle for the provability predicate that looks like Disjunctive Syllogism; and for a dialetheist it needn't be provable that not both A and ¬A are provable.]



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Curry Paradox Check List

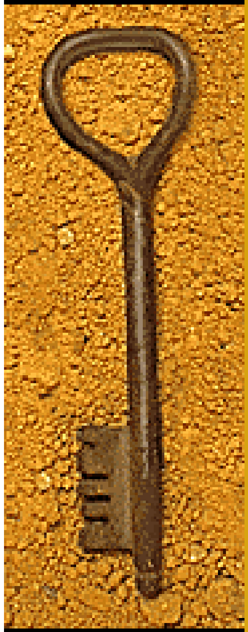
- In the light of the Curry Paradoxes we get into trouble if the four following conditions are met simultaneously:

1. Convention (T) and/or Naive Comprehension hold.

2. Absorption/Contraction as theorem: $(A \supset (A \supset B)) \supset (A \supset B)$
or Absorption/Contraction as a rule: $A \supset (A \supset B) \vdash (A \supset B)$
or the theorem version of *Modus Ponens* and furthermore *either* $\vdash (A \wedge A \equiv A)$ *or* $\vdash (A \wedge A \supset B) \Rightarrow \vdash (A \supset B)$
or a Cut *and* a Conditionalization Rule.
or $\vdash (A \supset A)$ and $\vdash (A \supset (B \supset C)) \supset ((A \supset B) \supset (A \supset C))$

3. Closure under substitution of proven equivalents
or $\vdash (B \equiv A), \vdash (B \supset C) \Rightarrow \vdash (A \supset C)$
and $\vdash (B \equiv A), \vdash B \Rightarrow \vdash A$
or $\vdash (C \wedge A) \Rightarrow \vdash A \;\&\; \vdash C$
or $\vdash (B \equiv A), \vdash (C \wedge A \supset D) \Rightarrow \vdash (C \wedge B \supset D)$

4. Closure under *Modus Ponens*.



Curry Paradox Check List (II)

- 4 seemingly cannot be given up because of the *Modus Ponens* condition.
- Cut/Transitivity is as closely tied to our intuitive understanding of consequence as is *Modus Ponens*.
- 1 cannot be given up in light of the aims of paraconsistency. (The interchange principle (TE) should also hold, it seems.)
- Further on 3 should not be given up lightly, since substitution of proven equivalents is a basic operation.
- Simplification of a conjunction is a basic property of conjunction.
- *Therefore* Absorption/Contraction in both its forms and the *Modus Ponens* theorem have to go (blocking the first three disjuncts in point 2), and Conditionalization has to be given up or restricted in some fashion [at least when occuring in the kind of meta-reasoning just considered].
    - at least so it seems.

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Giving Up on Contraction/Absorption

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- In the first version of the Curry Paradoxes we have used the basic case of Contraction:

  $$(A\supset(A\supset B)) \supset(A \supset B)$$

  Contraction can occur with more nested conditionals, like

  $$(A\supset((A\supset(A\supset B)) \supset(A \supset B))$$

  Such a longer version could, of course, also be used for trivialization by using one more application of *Modus Ponens*.
- What really needs to be avoided is thus, so to say, *Generalized Contraction*.
- It is also not crucial that the conditional connective itself shows the contractive behaviour. Trivialization occurs also if there is a weaker connective "$>$" (implied by "$\supset$") which fulfils these conditions:

  $$A \supset B \vdash A > B$$

  $$A, A > B \vdash B$$

  $$A > (A > B) \vdash A > B$$

The *Modus Ponens* Condition – Reconsidered

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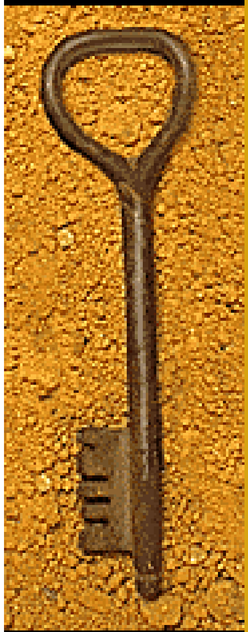
- The *Modus Ponens* Condition seems to be obvious. What else should a conditional be than a connective observing *Modus Ponens*.
- But there are problems with sticking strictly to this condition:
  (a) Cut *and* Conditionalization seem also very natural for a conditional connective,
  (b) Giving the dialetheist goal to supersede the object-/meta-language distinction, it is not that easy to keep the *Modus Ponens Theorem* and *Modus Ponens* apart: One should be able to express *in a sentence of our language* that *Modus Ponens* holds. This sentence would look rather like some version of the *Modus Ponens Theorem*. On pains of coming close to the ineffability of the validity of *Modus Ponens*, there should be something like a *Modus Ponens Theorem*.
- Going back to different language levels to express the validity of *Modus Ponens* for the conditional connective of the next lower level brings back hierarchy [cf. (Whittle 2004)] and so MYSTERY.

The *Modus Ponens* Condition – Reconsidered (II)

- We can avoid these problems by – *giving up* the idea of a *Modus Ponens* that holds under all conditions imaginable.
- This seems devastating (and contrary to everything said before on behalf on *Modus Ponens*), but it only seems so:
  1. That there are exceptions to *Modus Ponens* does not mean that it does not hold almost completely; there is a – vague but obvious – distinction between gross violations of the *Modus Ponens* Condition (e.g. having no detachable conditional connective) and a rule of *Modus Ponens* that allows for some *principled exceptions* only.
  2. We will consider versions of paraconsistent logics that have the virtue of explaining why and where *Modus Ponens* may fail [cf. Chap. 7 and 20]. We may reconsider Cut in this context as well.
  3. That we do not use detachment related reasoning in the vicinity of Explosion was one of the main starting points of paraconsistency.
  4. Once one allows for some exceptions to *Modus Ponens*, the distinction between object- and meta-language can be dropped, and the *Modus Ponens Theorem* should not be valid anyway.



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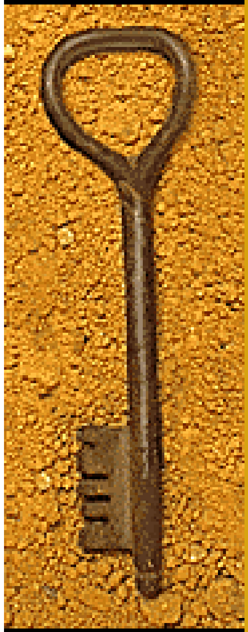
Some Failures



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- Judging from the list of criteria given some paraconsistent approaches are failures (whatever else their worth for pure logical investigations may be). One should stress here that there are at least two motivations to do logic: (a) the philosophical one, (b) the pure logical one.
- (ad a)
  We follow here both philosophical and logical aims, that is the whole point of having the set of criteria to be met; these criteria articulate philosophical goals (like semantic closure) and methodological aims (like keeping close to our ordinary intuitions).
- (ad b)
  Logics can be studied from a purely abstract point of view to see which systems are possible, tenable, and how they are related to each other. The systems may be smooth, but of no immediate use. There is nothing wrong about this, it just isn‘t *philosophical* logic proper.
- The approaches we look at are selected either because they are prominent, even if they do not comply with the criteria given, or because they are promising attempts to develop PLs fulfilling the criteria (some versions of them may still not satisfy the criteria).

p ∧¬p

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Jaskowski Systems

- Jaskowski was a pioneer of paraconsistency. His logic was thought to deal with drawing conclusions from a debate where several people have different opinions on a matter and so contradict each other. Although he did not see this system as a general PL, we will do him here some injustice – for methodological reasons only – to give an example of an approach that does not meet the criteria developed.
- The logic is a modal logic. Each possible world represents a speaker and what she asserts (as true). Some sentence A is true in a model *M* if there is at least one speaker's world in which A is true:

  (1) $M \models_d A \Leftrightarrow$ there is w in M, such that $w \models_d A$

  Consequence and validity are defined accordingly:

  (2) $\models_d A \Leftrightarrow (\forall M)(M \models_d A)$

  (3) $\Gamma \models_d A \Leftrightarrow (\forall M)((\exists B \in \Gamma) \neg(M \models_d B) \vee (M \models_d A))$

  A is a consequence of Γ iff either not all premises are true or A is true. Note that the premises are true iff they are true in *one* world of M.
- The valid sentences of Jaskowski's system are the theses of the modal logic **S5**: $\vdash_d A \Leftrightarrow \vdash_{S5} A$ [cf. Jaskowski 1969]. So *all* Irrelevant theses (like the paradoxes of strict implication) are valid!

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Jaskowski Systems (II)

- To avoid Explosion it must not be valid:
  (*4) $\{A, \neg A\} \models_d B$
  If two people contradict each other in a debate that doesn‘t mean that they now believe everything. That two contradictories are held true in a debate does neither imply that there is a single speaker who believes both nor that the debating group believes their conjunction. So it has to be invalid:
  (*5) $\{A, B\} \models_d (A \wedge B)$
  ($\wedge$I) has to be rejected! Therefore this type of logic is called "non adjunctive". This is a severe violation of the Extensionality Condition.
- In fact there is *no* recursive truth condition Ψ for "$\wedge$" such that
  (*6) $M \models_d (A \wedge B) \Leftrightarrow \Psi(M \models_d A, M \models_d B)$
  since in a real debate *some* sentences are joint and *some* are not, i.e. we can have in a model *M:* $M \models_d A$, $M \models_d B$, $M \models_d C$, $M \models_d (A \wedge B)$, and *not* $M \models_d (A \wedge C)$.
- Rejecting ($\wedge$I), however, does not make sense in modelling a debate either: Sometimes it is useful to assert which contradictions (between speakers) have arisen in a debate, and what follows from *that*.

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Jaskowski Systems (III)

- Even though Explosion in the form of (*4) is blocked, Explosion in the form $\{A \wedge \neg A\} \models_d B$ is valid given that all S5-thesis and *Modus Ponens* hold.
- Jaskowski, therefore, introduced a discussive implication:

  (7) $\quad A \supset_d B := \Diamond A \supset B$

  This implication fulfils the <u>Modus Ponens Condition</u>, since in system **S5** we have

  (8) $\quad A \supset \Diamond A$
- But because of (8) the fact obtains that any sentence generated from a tautology of **PC** by substituting "$\supset_d$" for "$\supset$" is valid in Jaskowski's system (including Absorption). This brings with it the violation of the <u>Strong Anti-Triviality Condition</u>.
- So it seems not to be too easy to develop some paraconsistent logic which satisfies all the criteria given.

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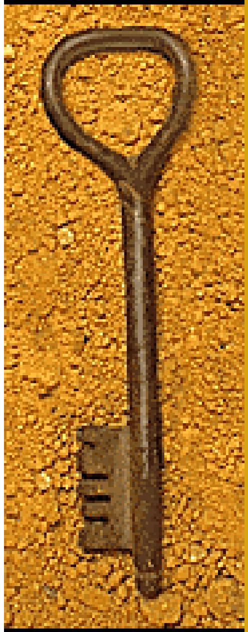
Halldén's Logic of Nonsense (**LN**)

- A major anticipation of some devices of paraconsistent logics occurs in Sören Halldén's book *The Logic of Nonsense*, published 1949.
- Halldén investigates not only paradoxes but also vague expressions and other sentences considered as unverifiable, and thus supposedly nonsensical. His solution to the paradoxes is a version of the many-valued approach (and thus fails). Halldén's logic **LN**, however,
  (i) contains *object-language* operators [+/–] to distinguish meaningful [+A] from nonsensical sentences [–A], thus anticipating one of the basic devices of the Logics of Formal Inconsistency [cf. Chap 8.].
  (ii) has not just one but *two distinguished truth values* [1, n] (of the three [1, 0, n] present in the system), thus anticipating the major device of the logic **LP** [cf. Chap. 4].
  (iii) has a version **LN+** with something like a *variable sharing requirement* for cases of detachment, thus anticipating one of the central ideas of Relevant Logics [cf. Chap. 5].
- Concerning meaningfulness Halldén considers any complex sentence as meaningless which contains a meaningless part. This leads to the weak Kleene truth tables of a 3-valued logic.



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Halldén's Logic of Nonsense (II)

A	**B**	**¬A**	**A∧B**	**A∨B**	**A⊃B**	**A≡B**
1	1	0	1	1	1	1
1	0	0	0	1	0	0
1	n	0	n	n	n	n
0	1	1	0	1	1	0
0	0	1	0	0	1	1
0	n	1	n	n	n	n
n	1	n	n	n	n	n
n	0	n	n	n	n	n
n	n	n	n	n	n	n

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Halldén's Logic of Nonsense (III)

- Now, since both *n* and 1 are designated truth values one has – in contrast to Kleene's weak system – still logical truths, namely *all standard taulologies* of **PC**. This is due to the fact that one may consider the value "either 1 or *n*" as a weak form of non-falsity and thus as a weak form of truth, yielding sentences never evaluated as 0.

  Meta-Theorem 1. $\vdash_{PC} A \Rightarrow \vdash_{LN} A$

- With respect to consequences matters stand completely different. Most of the standard consequences are not valid in **LN**. This makes the system paraconsistent, since *ex contradictione qoudlibet* doesn't hold:

  $A, \neg A \nvdash_{LN} B$

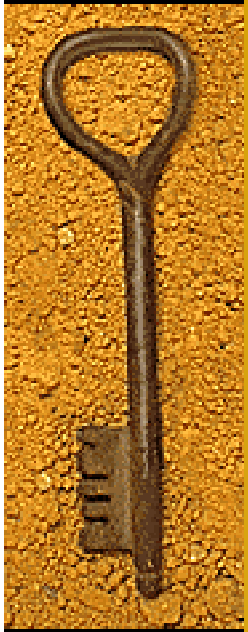
  $A \wedge \neg A \nvdash_{LN} B$

  because if A receives the value *n*, ¬A also receives the value *n* thus both premises or the conjunction having a designated truth value with B open for any value, especially 0.

- **LN** is not only paraconsistent, but also avoids Curry Paradoxes by invalidating *Modus Ponens* as a rule:

  $A, A \supset B \nvdash_{LN} B$

  because if A is nonsensical both A and A ⊃ B are designated, even if B is false.



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Halldén's Logic of Nonsense (IV)

- *Meta-Theorem 2*. **LN** is non-trivial.

  $$\nvdash_{LN} +B$$

  i.e. not every sentence is meaningful.
- Although avoiding Curry Paradoxes and fulfilling the Weak Anti-Triviality Condition **LN** thus fails on the Modus Ponens Condition. Because of the variable sharing requirement in **LN+** *Modus Ponens* holds in all cases fulfilling the requirement.
- Matters are far worse, however. Because any compound with a nonsensical part receives a designated truth value it is very easy to have the premises of a supposed consequence designated and the supposed conclusion being false. Almost all standard consequences – even the elimination rules of the extensional connectives – fail!
- For example:

  $$A \wedge B \nvdash_{LN} B$$

  because if A is nonsensical the conjunction is designated *even if* B is false.
- This is an unacceptable failure on the Minimal Damage and the Extensionality Condition.

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Halldén's Logic of Nonsense (V)

- The damage can be partly undone when we look at the *meaningful* formula, since these should behave as **PC** formula do.
- Thus we have

  $$+(A \wedge B) \vdash_{LN} B$$

  because the conjunction can only be meaningful if both parts are meaningful, and such a meaningful conjunction is true only if both parts are true.
- In this way standard consequences can be recaptured. This applies, however, also to the consequence a paraconsistent system would like to avoid. So one has

  $$+A, A, \neg A \vdash_{LN} B$$

  $$+(A \wedge \neg A) \vdash_{LN} B$$

  since if A is meaningful it is not possible that A and ¬A are both true, so *ex contradictione qoudlibet* holds!
- This dilemma/bifurcation we will see again with LFI-systems.
- The distinction between +A and A useful as it is also presupposes that in the process of formalization we already know whether a formula is meaningful or not.

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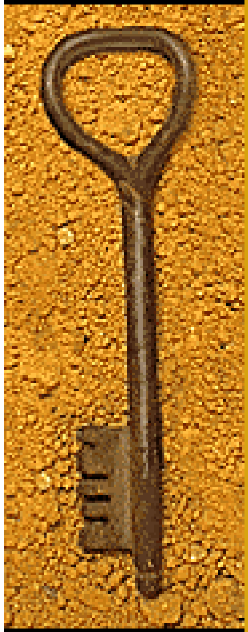
Questions

- (Q1) That $A \supset \Diamond A$ secures the obtaining of the usual detachment conditions in Jaskowski's system holds only if we keep another property of the conditional. Which one?
- (Q2) The sentence "$p \wedge (p \supset q) \supset q$" is called *Modus Ponens Theorem*. Is that a good name?
- (Q3) If one was to introduce a new intensional conjunction into Jaskowski's system, a conjunction which satisfies ($\wedge$I), why would this step still result in a system that violates the Extensionality Condition?
(Think of the relation between the connectives.)



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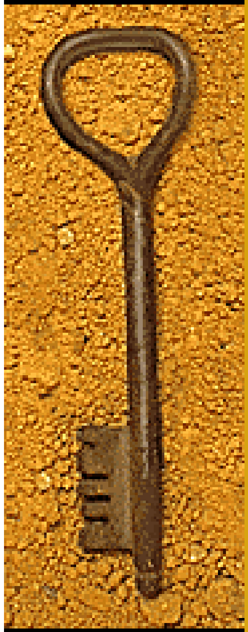
Exercises

- (Ex1) *Peirce's Law* is the following conditional:

  $$((A \supset B) \supset A) \supset A$$

  One may define a Curry Sentence $x \in x \supset B$, and thus a Curry Set $C = \{x | x \in x \supset B\}$.
  With Naïve Comprehension we have:
  $(C \in C \supset B) \equiv (C \in C)$
  Give a Curry Paradox using *Modus Ponens*, Naïve Comprehension, and Peirce's Law.
- (Ex2) Which of the standard natural deduction rules ($\wedge$E), ($\neg$E), ($\supset$E), ($\vee$E), ($\wedge$I), ($\neg$I), ($\supset$I), ($\vee$I) hold in **LN**?



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Further Reading

- On giving up transitivity of provable implication see: Tennant, Neil. *Anti-Realism and Logic*. Oxford, 1987. On Tennants work see (Bremer 1998, pp.59-66) and on the incompleteness of Tennant‘s system and the metalogical validity of Explosion see (Milne 1996).
- On the Curry Paradoxes see: Meyer, Robert/Routley, Richard/Dunn, Michael. "Curry's Paradox", *Analysis*, 39 (1979), pp. 124-28.
- On giving up Contraction and different routes to trivialization see: (Rogerson/Restall 2004) and Restall, Greg. “How to Be *Really* Contraction Free”, *Studia Logica*, 52 (1993), pp. 381-91; also: Bunder, M.W. "Tautologies that, with an Unrestricted Comprehension Axiom, Lead to Inconsistency or Triviality", *Journal of Non-Classical Logic*, 3 (1986), pp.5-12.
- The purely semantic approach is followed in Rescher, N./Brandom, R. *The Logic of Inconsistency*. Oxford, 1980.

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Further Reading (II)

- Halldén’s book *The Logic of Nonsense* appeared 1949 in Uppsala. Halldén is usually not mentioned as a forerunner of paraconsistency, and his philosophical theses on the paradoxes and meaning seem *quite* peculiar. Halldén calls his logics **C** and **CC**. Besides the standard tautologies these system contain further theorems involving the meaningfulness operators [like: $+(p \supset q) \supset +p \wedge +q$]. Looking at these may be of interest in comparing this precursor systems to today’s LFI-systems. From a historical perspective the book is worth looking at.
- On Jaskowski see: Urchs, Max. "Discursive Logic: Towards a Logic of Rational Discourse", *Studia Logica*, 54 (1995), pp. 231-49, and Urchs, Max. "On the Role of Adjunction in Para(in)consistent Logic", in: (Carnielli/Coniglio/D’Ottavanio 2002), pp. 487-500.

  [Urchs defends the Jaskowski approach as one being unjustifiably "bashed" within the PL community. The present chapter might be considered one more instance of that bashing – the only question is, in the light of which criteria the system is rejected; in other contexts the criticism employed here may not apply.]