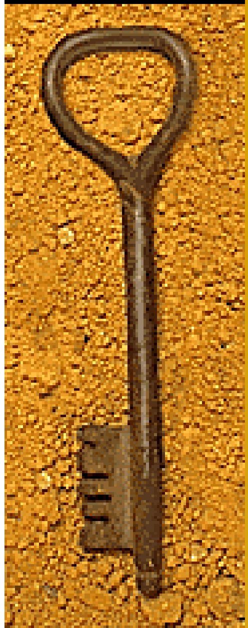
LFI and da Costa Systems



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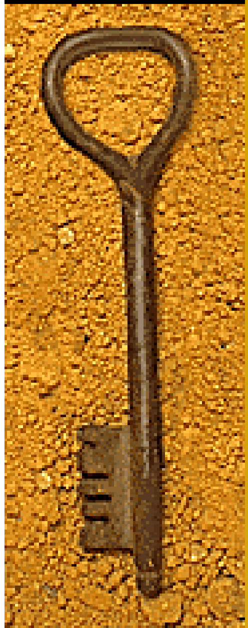
- Newton da Costa was one of the first to invent paraconsistent logics. He developed a hierarchy of so called *C-Systems*, logics which are inconsistency tolerant, but may have explosion with respect to some sentences. Working together with a couple of visiting researchers he applied these systems to problems of science and mathematics.
- Starting from da Costa's work the Brazilian approach to paraconsistency was established, and is today one of the main pillars of the paraconsistent world. Several researchers use C-Systems as their preferred paraconsistent logic.
- In the honour of Newton da Costa the 2nd World Congress on Paraconsistency was held in Brazil in 2000.
- Da Costa's work and the Brazilian approach has been extended and systematized by Walter Carnielli and his co-workers. They developed the more general frame work of *Logics of Formal Inconsistency* (LFI), into which the C-Systems can be integrated as one variant.
- This chapter follows the LFI approach and thereby introduces daCosta-Systems on the way.



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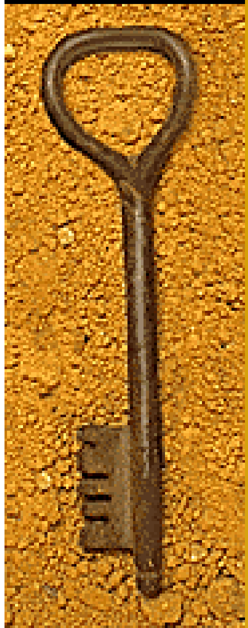
LFI-Systems Defined

- *Logics of Formal Inconsistency* (LFI) are paraconsistent logics that contain within themselves ways to express consistency and inconsistency (i.e. within the object language), introducing operators. Consistency is taken as a *primitive* notion and expressed by "°", "°A" says that A is consistent, i.e. we do not have A and ¬A.
  [cf. Chap. 4 on expressing truth values in **LP**, and on "°A" and "•A"].
- *C-Systems* are those LFIs that are extensions of the positive part of some consistent logic (like PC). The behaviour of negation and in which ways one wants to have Explosion, Contraposition, Excluded Middle etc. distinguishes different C-Systems.
- DaCosta in introducing his systems wanted to combine paraconsistent reasoning with the strength of standard logic in restricted contexts. Although paraconsistent reasoning is non-explosive, explosion should, according to daCosta, be valid in contexts considered consistent. So some formula was taken to *express the consistency* of a formula A: $\neg(A \wedge \neg A)$. A formula considered consistent in this way should exhibit explosive behaviour in case one had A and ¬A! DaCosta-systems in the narrow sense are systems *in which* "°" is *definable* thus.



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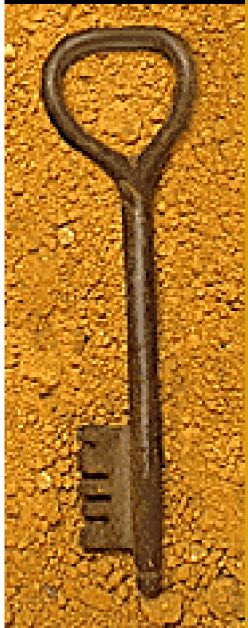
Principles

- The LFI-approach distinguishes inconsistency from contradiction. A logic respects the *Principle of Non-Contradiction* (PNC) if it has non-contradictory theories (i.e. sets of sentences closed under that logic that do either not contain A or not ¬A). This requires that the logic itself has not contradictions as *theses* (as some version of naive semantics may have). A logic is contradictory if all of its theories are.
- A logic respects the *Principle of Non-Triviality* (PNT) if it is not trivial. $(\exists \Gamma)\ (\exists B)(\Gamma \nvdash B)$
- A logic respects the *Principle of Explosion/Pseudo Scotus* (PPS) if it is explosive. All paraconsistent logics disrespect (PPS).
- A logic respects the *Principle of Ex Falso Qoudlibet Sequitur* (ExF) if it has some formula that can trivialize the logic by itself:

  $(\exists C)\ (\forall \Gamma, B)(\Gamma, C \vdash B)$
- A logic respects the *Supplementing Principle of Explosion* if it (also) has a strong negation "~" for which Explosion holds.



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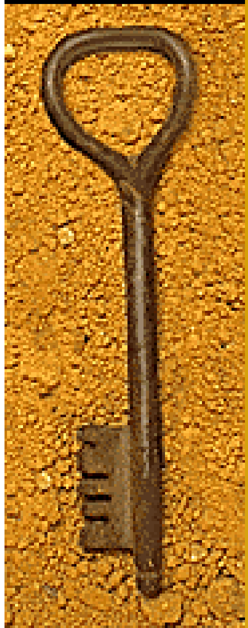
Being *Gently Explosive*

- LFIs are "gently explosive" in the vain of daCosta's idea:

  (gPPS) A theory Γ is *gently explosive* when there is a way of expressing the consistency of a formula A by way of formulas which depend only on A and which cannot be added to Γ together with $\{A, \neg A\}$ without yielding triviality.
- So LFIs distinguish between fulfilling (PPS) and being explosive in some way, i.e. fulfilling (gPPS).
- In this context one may investigate whether a logic allows to define or to add a bottom particle "$\perp$" that allows to conclude everything.
- Further on one may introduce different negations. A paraconsistent negation " $\neg$ " disrespecting (PPS) and a "strong" negation "~" respecting (PPS).



" ∘ " and " • "

- The consistency and the inconsistency operator can either be introduced into a system as primitive or as being defined by some formula.
- If "°A" is taken as defined by $\neg(A \wedge \neg A)$ it says that only one of these obtains, so is a hidden *disjunction*. It does *not* say that A obtains!
- If " •A" is taken as defined by $(A \wedge \neg A)$ it is a hidden *conjunction*.
- In case the operators are syntactic primitives their semantics may be given by a truth table:

A	°A	•A
1	1	0
0	1	0
0,1	0	1

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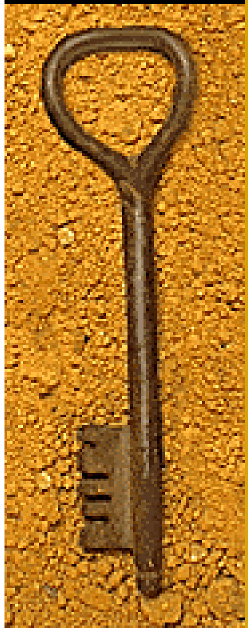

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LFI 1

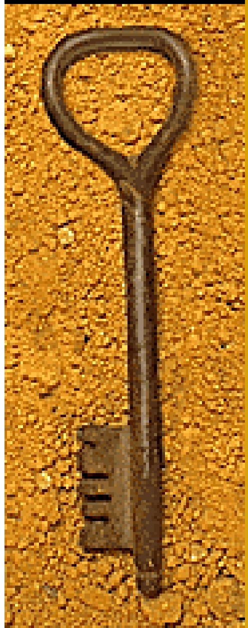
- There is a plenty of LFI systems. (In fact because of combinatorial reasons related to the choice of inheriting consistency "°" over connectives like "$\wedge$" or "$\vee$" there are *thousands* of different systems.)
- The LFI-approach tries to incorporate into the field of LFIs also systems like **Pac** or $\mathbf{J_3}$ that were independently developed.
- Before we look at the syntactic generation of successively stronger LFI systems let us take a look at a basic truth functional semantic presentation.
- **LFI 1** (also called "**Pac**" in case "°" is missing, also called "**J3**" if additionally a strong negation and a bottom particle are present) is given by the following truth tables.

A strong negation is not *definable* within **LFI 1**, but can be added.

[To keep a more or the less coherent formalization in these lectures the conditional connective is here represented as "$\supset$" to distinguish it from that of the Relevant Logics and from **SKP**. The truth value of inconsistent formulas is again given as "0, 1" and not as "½".]

LFI 1 Tables

A	**B**	**¬A**	**A∧B**	**A∨B**	**A⊃B**	**°A**
1	1	0	1	1	1	1
1	0	0	0	1	0	1
1	0,1	0	0,1	1	0,1	1
0	1	1	0	1	1	1
0	0	1	0	0	1	1
0	0,1	1	0	0,1	1	1
0,1	1	0,1	0,1	1	1	0
0,1	0	0,1	0	0,1	0	0
0,1	0,1	0,1	0,1	0,1	0,1	0



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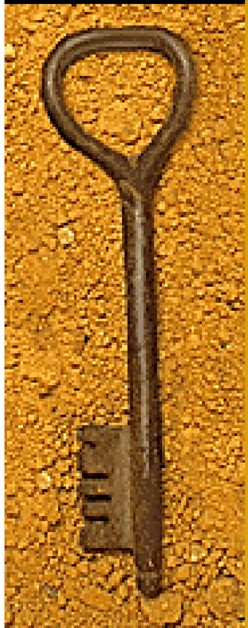
The Axiomatic Basis of LFI Systems



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- The Axiomatization of LFI systems can start from this basis:

  (A1) $p \supset (q \supset p)$
  (A2) $(p \supset q) \supset ((p \supset (q \supset r)) \supset (p \supset r))$
  (A3) $p \supset (q \supset p \wedge q)$
  (A4) $p \wedge q \supset p$
  (A5) $p \wedge q \supset q$
  (A6) $p \supset p \vee q$
  (A7) $q \supset p \vee q$
  (A8) $(p \supset r) \supset ((q \supset r) \supset ((p \vee q) \supset r))$
  (A9) $p \vee (p \supset q)$
  (A10) $p \vee \neg p$
  (A11) $\neg \neg p \supset p$

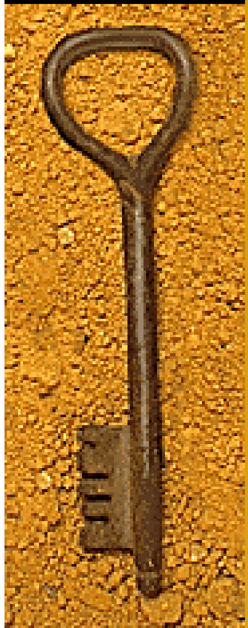
  Rules: (MP) and *Uniform Substitution.*
  [Note that by (A1) these systems are not Relevant, by (A10) they are anti-intuitionistic; cf. Batens' **CLuN** in Chap. 7]

- Given (A1) and (A2) the *Deduction Theorem* holds here:

  $\Gamma, C \vdash B \Rightarrow \Gamma \vdash (C \supset B)$

The Axiomatic Basis of LFI Systems (II)

- This is indeed the basis of paraconsistent logics:
  Adding
  $\neg p \supset (p \supset q)$
  would yield standard PC, but this formula is not valid, as can be seen by the **LFI 1** truth tables.
- This basis does neither have a strong negation nor a bottom particle.
- This system has not a negated theorem: $\neg\,(\exists A) \vdash (\neg A)$
  Thus: $\nvdash (\neg\,(p \wedge \neg p))$
  The *Law of Noncontradiction* is no theorem here!
- [(A1) – (A8) is usually called "positive logic". (A9) is a positive formula, but not provable from (A1) – (A8).]



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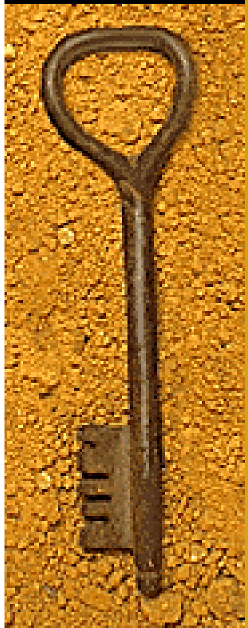

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The System **bC**

- Adding to the basis the following rule gives the system **bC**:

  (bc1) $°A, A, \neg A \vdash_{\mathbf{bC}} B$

  bC contains a primitive connective "°" and this rule of gentle explosion. This leads to a conservative extension of the basis logic.
- Given "°" a strong negation and a bottom particle can be defined:

  (D~) $\sim A := \neg A \wedge °A$

  (D⊥) $\bot := \sim A \wedge A$

  since Explosion is valid for strong negation:

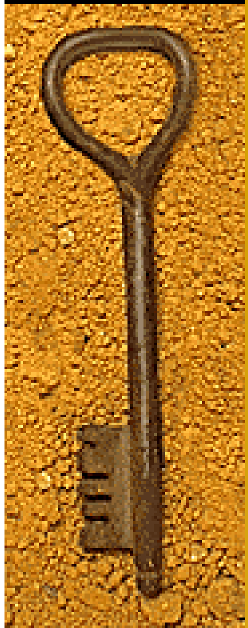
  $A, \sim A \vdash_{\mathbf{bC}} B$
- In contrast to be basis **bC** does have negated theorems

  $\vdash_{\mathbf{bC}} (\neg \bot)$

  and equivalent negated formulas,

  but no theorems of the form °A, i.e. even:

  $\nvdash_{\mathbf{bC}} °(p \vee \neg p)$



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The System **bC** (II)

- The following rules but *not their converses* hold in **bC**.

  (i) $A, \neg A \vdash_{\mathbf{bC}} \neg{\circ}A$ (ii) $A \wedge \neg A \vdash_{\mathbf{bC}} \neg{\circ}A$

  (iii) ${\circ}A \vdash_{\mathbf{bC}} \neg(A \wedge \neg A)$ (iv) ${\circ}A \vdash_{\mathbf{bC}} \neg(\neg A \wedge A)$

  [This can be shown by changed **LFI 1** truth tables that take "¬" from "0,1" to "0", " ∨" from "0" and "½" to "1", and " ⊃" from "1" and "½" to "1". Since these tables are *sound* for **bC**, but invalidate the converses of (i) – (iv), these cannot be *provable*; this procedure is used for all results of this type; the deviant tables may not have an intuitive reading, their sole purpose is to prove the deductive weakness of the system in question.]

- The dualities for the connectives usually do *not hold* in **bC**, e.g.:

  $\neg(p \wedge q) \nvdash_{\mathbf{bC}} \neg p \vee \neg q$ [neither the converse]

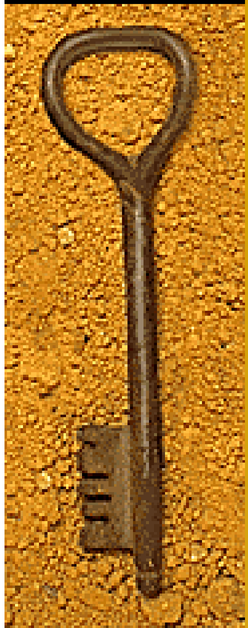
  $\neg(\neg p \vee \neg q) \nvdash_{\mathbf{bC}} (p \wedge q)$ [neither the converse]

  $p \vee q \nvdash_{\mathbf{bC}} \neg p \supset q$ [converse holds]

  $p \supset q \nvdash_{\mathbf{bC}} \neg(p \wedge \neg q)$ [neither the converse]

  $\neg (p \supset q) \nvdash_{\mathbf{bC}} (p \wedge \neg q)$ [neither the converse]

- Contraposition does not hold: $(p \supset q) \nvdash_{\mathbf{bC}} \neg q \supset \neg p$ otherwise by (A1) and (MP) **bC** would turn out to be explosive! In fact, adding this form of Contraposition would yield PC.



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The System **bbbC**

- In **bC** we have "°", but no corresponding axioms or theorems concerning its dual "•". There are four rules of duality:

  (bc2) $\neg\, {}^{\bullet}A \vdash {}^{\circ}A$

  (bc3) $\neg\, {}^{\circ}A\, {}^{\bullet}A \vdash \neg\, {}^{\circ}A\, {}^{\bullet}A$

  (bc4) ${}^{\bullet}A \vdash \neg\, {}^{\circ}A$

  (bc5) ${}^{\circ}A \vdash \neg\, {}^{\bullet}A$

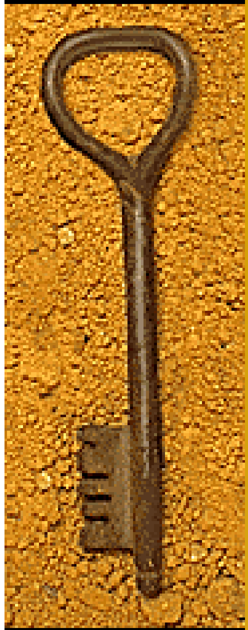
- In **bC** neither (bc2) nor (bc3) hold. The addition of (bc2) does, by the absence of Contraposition, not yield (bc3). Adding (bc2) and (bc3) to **bC** gives the system **bbC**. In this again neither (bc4) nor (bc5) are provable, which again are independent of each other. Adding (bc4) and (bc5) to **bbC** yields **bbbC**.
- In **bbbC** the relation between °A and ¬ •A *cannot* be characterised by the intuitive definition:

  (D•) ${}^{\bullet}A := \neg\, {}^{\circ}A$

  since then by (bc5) and straightforward substitution we would have

  $\vdash ({}^{\circ}p \supset \neg\neg\, {}^{\circ}p)$

  which is not valid in **bbbC**! The converse of (A11) does not hold.



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The System **Ci**

- In **bbbC** we have for "$\bullet$" not its intuitive definition. One should expect that $^{\bullet}$A, meaning that A is contradictory, has a corresponding syntactic characterization. This is given in the system **Ci**.
- **Ci** is **bbbC** supplemented by the rule

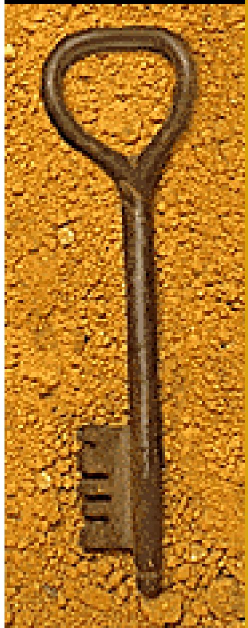
  (ci) $\quad {}^{\bullet}A \vdash (A \wedge \neg A)$

  So in **Ci** we have the inconsistency operator " $\bullet$" being characterized by formulas, but we don't have: $\neg (A \wedge \neg A) \vdash {}^{\circ}A$.
- One could give **Ci** also by using one of the intuitive definitions

  (D$\bullet$) $\quad {}^{\bullet}A := \neg\, {}^{\circ}A$

  or (D$^{\circ}$) $\quad {}^{\circ}A := \neg\, {}^{\bullet}A$

  and add this to **bC** and (ci). We get then (bc2) – (bc5).



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The System **Ci** (II)

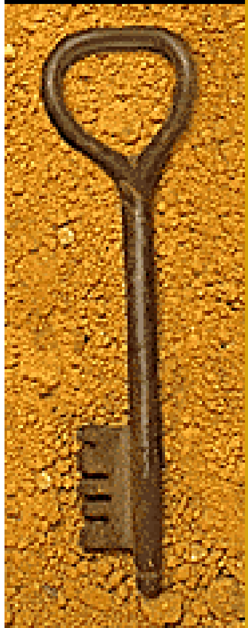
- In **Ci** we have:

  (DR1) ${}^\circ A, {}^\bullet A \vdash B$
  (DR2) ${}^\circ A, \neg {}^\circ A \vdash B$
  (DR3) ${}^\bullet A, \neg {}^\bullet A \vdash B$
  (DR4) ${}^\circ A \vdash {}^\circ \neg A$
  (DR5) ${}^\bullet \neg A \vdash {}^\bullet A$
  (T1) $\vdash {}^\circ {}^\circ p$
  (T2) $\vdash {}^\circ {}^\bullet p$
  (T3) $\vdash (p \vee {}^\circ p)$

- A particular schema in **Ci** is consistent if and only if we have controlled explosion with that schema in **Ci**:

  $$\Gamma \vdash_{Ci} {}^\circ A \Rightarrow \Gamma, A, \neg A \vdash_{Ci} B$$

- If one added Contraposition as a rule to **Ci** one would regain PC.
- Axiom (A9) is redundant in **Ci**.



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The System **Cil**

- In **Ci** we could identify inconsistency (expressed by "•") with contradictoriness by

  (ci) $\bullet A \vdash (A \wedge \neg A)$

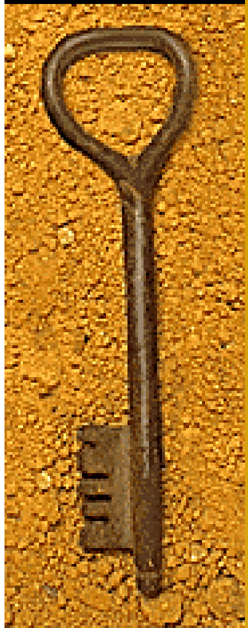
  We could not derive consistency (expressed by "°") from the negation of contradictoriness, since we have:

  (*) $\neg (A \wedge \neg A) \nvdash_{Ci} {\circ}A.$
- If we add to **Ci** the scheme

  (cil) $\neg (A \wedge \neg A) \vdash {\circ}A.$

  we get the logic **Cil** which identifies contradictoriness with inconsistency, daCosta's original approach. (D•) and (D°) can be used.
- From (cil) it is obvious – since otherwise simple Explosion would be back – that from **Cil** on no LFI-system can have $\neg(A \wedge \neg A)$ as a theorem, *except* substitution of provable equivalents fails in these systems.
- The consistency of A can be expressed in **Cil** by $\neg(A \wedge \neg A)$, but *not* by $\neg(\neg A \wedge A)$! The latter can even be added to **Cil** without it loosing its paraconsistent character: $\neg(\neg A \wedge A) \nvdash_{Cil} \neg(A \wedge \neg A)$.

Propagating Consistency



- In **Cil** we could propagate consistency to negation by

  $$°A \vdash_{Cil} °\neg A$$

- How then can consistency be propagated from subformula to complex formula? A natural assumption seems to be that the consistency of each subformula is sufficient for the consistency of the complex formula, thus

  (ca1) $(°A \wedge °B) \vdash °(A \wedge B)$

  (ca2) $(°A \wedge °B) \vdash °(A \vee B)$

  (ca3) $(°A \wedge °B) \vdash °(A \supset B)$

- The logic obtained by adding (ca1) – (ca3) to **Cil** is **Cila**.
- We have by (T1) of subsystem **Ci**, (D°), (ca1) and (cil):

  $$\vdash_{Cila} °\bot$$

  whatever that is supposed to mean! [Remember: $\bot \equiv °p \wedge (p \wedge \neg p)$]

- [Note: daCosta's hierarchy of systems is based on **Cila** (his $\mathbf{C_1}$); each system in the hierarchy of ever weaker systems modifies the definition of consistency of the previous one with nested occurences of "°"; cf. (daCosta 1974)]

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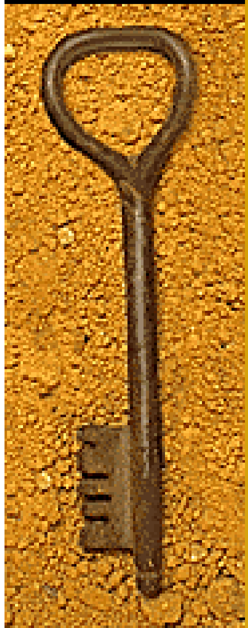

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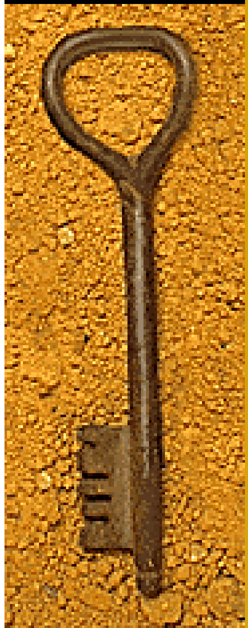
Propagating Consistency Back

- In **Cila** we could propagate from the subformulas to the formula. What about the other direction? One might have it that the consistency of the formula requires the consistency of all its subformulas – thus having the converses of (ca1) to (ca3) – or one may be satisfied with one of the subformulas being consistent, i.e. $°(A \wedge B) \vdash °A \vee °B$ etc.
- There is a plenty of options, and thus of LFI-systems.
- One may also start not with **Cila**, but with the weaker assumptions that the consistency of *one* subformula is sufficient for the consistency of the complex formula, i.e. $(°A \vee °B) \vdash °(A \supset B)$ etc.
  The corresponding system is **Cilo**.
- If supplementary to these options one toys with the options how the value (0,1) in a complex formula is related to the values of its subformula, keeping the compositionality of consistent formulas as in PC, one has a choice of about 8000 (in words: *eight thousand*) different LFI-systems! All these systems extend **Ci** and are subsystems of **ePC**.



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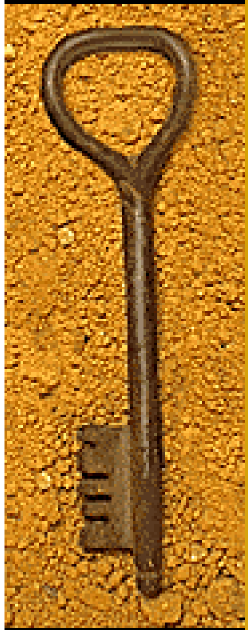
LFI 1 Syntactically

- **LFI 1** characterized by truth tables before can now be characterized syntactically. It is the system extending **Ci** with the rules:

(ce) $A \vdash \neg\neg A$

(cj1) $\bullet(A \wedge B) \vdash (\bullet A \wedge B) \vee (\bullet B \wedge A)$
(cj2) $\bullet(A \vee B) \vdash (\bullet A \wedge \neg B) \vee (\bullet B \wedge \neg A)$
(cj3) $\bullet(A \supset B) \vdash (A \wedge \bullet B)$

- So in **LFI 1** the propagating concerns inconsistency instead of consistency. (cj1) – (cj3) can be read off the truth tables.
- Remember that (D°) is not part of **LFI 1**, since (LNC) holds as a theorem in both forms (i.e. with a commuted conjunction); otherwise the order of conjuncts in **LFI 1** may have importance [see (cj1)].



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LFI 2

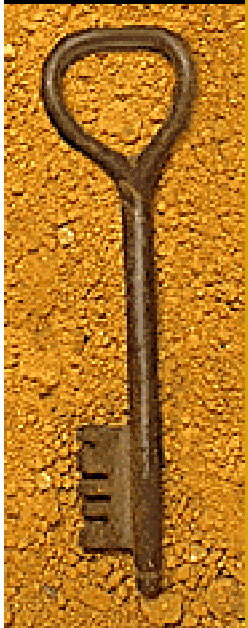
- **LFI 2** is the system extending **Ci** with the rules:

(ce) $A \vdash \neg\neg A$

(co1) $(°A \vee °B) \vdash °(A \wedge B)$
(co2) $(°A \vee °B) \vdash °(A \vee B)$
(co3) $(°A \vee °B) \vdash °(A \supset B)$

(cr1) $°(A \wedge B) \vdash °A \vee °B$
(co2) $°(A \vee B) \vdash °A \vee °B$
(co3) $°(A \supset B) \vdash °A \vee °B$

- Note that in **LFI 2** (cil) is missing, i.e. the consistency operator is not defined by some formula; thus **LFI 2** is not a daCosta-system.



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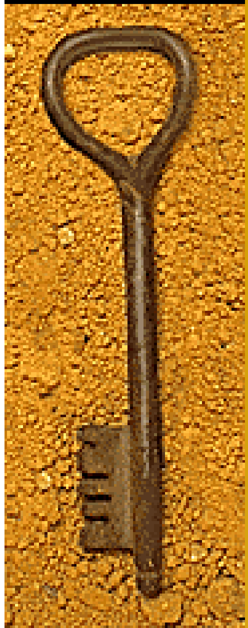
LFI 2 (II)

The **LFI 2** truth tables are:

A	**B**	**¬A**	**A∧B**	**A∨B**	**A⊃B**	**°A**
1	1	0	1	1	1	1
1	0	0	0	1	0	1
1	0,1	0	1	1	1	1
0	1	1	0	1	1	1
0	0	1	0	0	1	1
0	0,1	1	0	1	1	1
0,1	1	0,1	1	1	1	0
0,1	0	0,1	0	1	0	0
0,1	0,1	0,1	0,1	0,1	0,1	0



$p \wedge \neg p$

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Restrictions on Equivalence

- Typically (proven) equivalents can be substituted for each other.
- There is a long list of equivalencies coming with PC. Many of these do not hold paraconsistently. (Famously "$A \wedge \neg A \equiv B \wedge \neg B$" cannot hold for a paraconsistent logic avoiding triviality: Every contradiction is different, so to say.)
- In LFI systems, however, there is *massive* failure of equivalencies and substitution of equivalencies:

  (i) in the basis no different negated formula are provably equivalent;

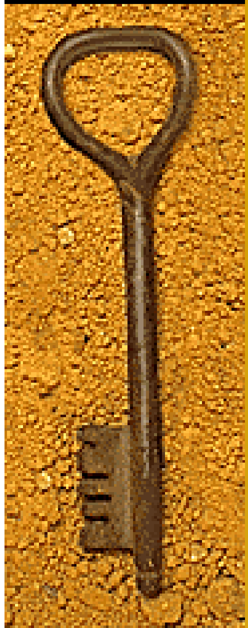
  e.g. $\neg(A \wedge B) \Leftarrow\!/\!\Rightarrow \neg(\neg(\neg A \vee \neg B))$

  (ii) in **bC** intersubstitutivity of provable equivalents *fails*:

  $(A \wedge B) \Leftrightarrow_{\mathbf{bC}} (B \wedge A)$, but $\neg(A \wedge B) \Leftarrow\!/\!\Rightarrow_{\mathbf{bC}} \neg(B \wedge A)$

  $(A \vee B) \Leftrightarrow_{\mathbf{bC}} (B \vee A)$, but $\neg(A \vee B) \Leftarrow\!/\!\Rightarrow_{\mathbf{bC}} \neg(B \vee A)$

  (iii) intersubstitutivity of provable equivalents fails also in **Ci** and *all logics* based on it, like **Cil** (on pains – as we have seen – on making these systems fall back into standard logic)!



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Expressing Standard Logic

- Typically LFI systems are *linguistic extensions* of PC, the set of well formed formula of PC being a proper subset of the well formed formula of some LFI.
- Given the – syntactic – expressive power of a language with a consistency operator "°" and an inconsistency operator " • ", possibly supplemented by a bottom particle "⊥" and a strong negation "~" it will be possible to *express the standard tautologies* in such a system.
- This is done by having a *translation* of standard formulas into the corresponding formulas of a LFI system.
- Those standard theorems and rules that are not paraconsistently valid have counterparts in the LFI system that are valid and which really express what the standard formula is thought to say. For example

  (1) $\neg p \wedge p \supset q$

  is not valid paraconsistently, but a gently explosive LFI may have:

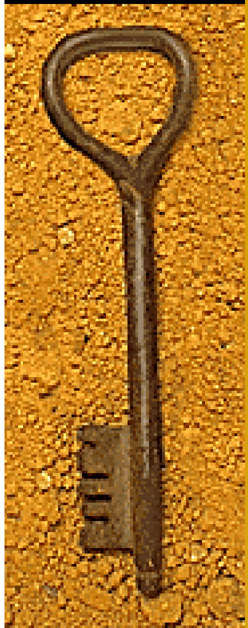
  (1') $°p \wedge (\neg p \wedge p) \supset q$
- One can uses an *extended* PC with °A having always value 1, and •A having always value 0. In PC proof theory one adds (D •) and the scheme: $\vdash °A$.

Expressing Standard Logic (II)

- Relative to *extended* PC [**ePC**] systems like **Ci** can be seen as subsystems of **ePC**.
- By translating PC-negation into the strong negation "~" of **Ci** one can have *all* the classical theorems expressed in **Ci**.
- The translation proceeds by a mapping:
  - (i) $t(A) = A$, for atomic A (e.g. $t(p)=p$)
  - (ii) $t(A\#B) = t(A) \# t(B)$ for any binary connective #
  - (iii) $t(\neg A) = \sim t(A)$
- As a result of the mapping this meta-theorem is true:

$$\Gamma \vdash_{\mathbf{ePC}} A \Longleftrightarrow t(\Gamma) \vdash_{\mathbf{Ci}} t(A)$$

where $t(\Gamma)$ is the application of the translation mapping to the formulas in Γ.



p ∧¬p

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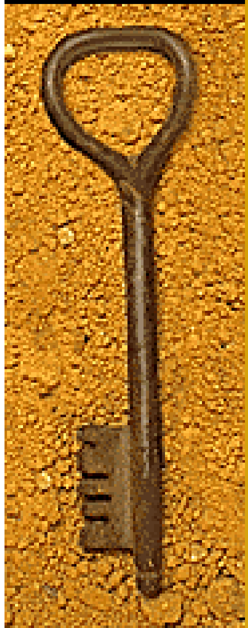

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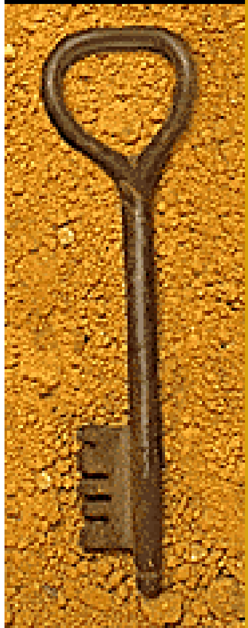
Assessment

- The thousands of LFI systems are a topic worth to be explored within pure logic of its own right.
- The philosophical motivation, however, to prefer some such system over another seems to be mostly missing. Especially the comparison to Relevant Logics, **LP** and **SKP** is not that favourable as it seems given the methodological constraints like the Extensionality Condition and the Minimal Damage Condition [in Chap. 3] – facing the failures of substitution and dualities of extensional connectives. These failures – on the other hand – ensure the non-triviality of Naive Set Theory or Naive Truth Theory given some LFI systems.
- The main criticism of **LP** that it is "an extremist position" in having no inconsistency being explosive, and commenting "that *consistency* is exactly what a contradiction might be lacking to become explosive", thus introducing "consistent contradictions" (Carnielli/Marcos 2002, pp. 26-28) awaits epistemic elucidation:
  If we have A and ¬A, then we should take °A as false, shouldn't we? And how can we *take* A to be consistent and have A and ¬A at the same time?



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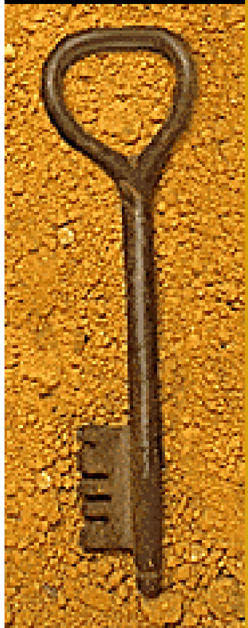
Assessment (II)

- The LFI-approach is, therefore, not the approach favoured here.

- Despite these criticisms the LFI-approach is one of the liveliest in the paraconsistent world, and turning to the literature surely requires knowing it.
- The idea of having primitive operators to express consistency and inconsistency is a useful idea that may lead to extensions of other systems [cf. Chap. 4 on the usage in **LP**].



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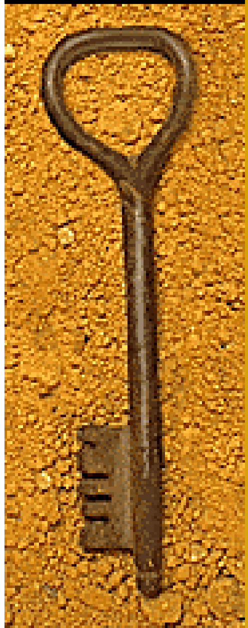
Questions

- (Q1) Which logic does not respect (PNT)?
- (Q2) Why does any non-trivial logic with a bottom particle and a conditional obeying the Deduction Theorem have a strong negation?
- (Q3) What distinguishes **LFI 1** from **LP** truth tables? And what consequences does this have?
- (Q4) Why isn't it contradictory to claim that **Ci** is a subsystem of **ePC** and nevertheless claim that *all* standard tautologies have *valid* translations in **Ci**?
- (Q5) What does: $\vdash_{\text{Cila}} \circ\bot$ say? Is it disastrous to have "$\circ\bot$"?
- (Q6) The **LFI 2** truth table for disjunction seems strange, doesn't it?



p ∧¬p

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Exercises

- (Ex1) Why can "$A \wedge \neg A \equiv B \wedge \neg B$" not hold in a typical PL with Conjunction Elimination? Show that it is not valid for some LFIs. In **LP** the formula is valid. Why is that not problematic? [Note that in **SKP** the rule (R7) of substitution requires provable mutual *entailment*; and: $\nvdash_{\mathbf{SKP}} (A \wedge \neg A \leftrightarrow B \wedge \neg B)$.]
- (Ex2) Show by **LFI 1** truth tables that $\neg A \supset (A \supset B)$ is not valid. (Is this different from **LP**?)
- (Ex3) Show by – deviant – truth tables:
  $\neg(A \wedge B) \nvdash_{\mathbf{bC}} \neg A \vee \neg B$
  $\neg(\neg A \vee \neg B) \nvdash_{\mathbf{bC}} (A \wedge B)$
  $A \vee B \nvdash_{\mathbf{bC}} \neg A \supset B$
  $A \supset B \nvdash_{\mathbf{bC}} \neg(A \wedge \neg B)$
  $\neg (A \supset B) \nvdash_{\mathbf{bC}} (A \wedge \neg B)$
- (Ex4) In **bbC** map " • " to truth value 1, and "°" to 0 in case of 0 and 1 and to (0,1) in case of (0,1). Show that (bc4) and (bc5) are not provable in **bbC**.

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Further Reading

- The presentation here is based on two comprehensive surveys of LFI and da Costa-Systems:
  - Carnielli, Walter/Marcos, Joao. "A Taxonomy of C-Systems", in: (Carnielli/Coniglio/Ottaviano 2002), pp. 1-94.
  - Carnielli, Walter/Coniglio, Marcelo/Marcos, Joao. "Logics of Formal Inconsistency", in: Gabbay, D./Guenthner, F. (Eds.) *Handbook of Philosophical Logic*, 2nd Ed., Vol. 12, 2004. This is available at:

  http://www.cs.math.ist.utl.pt/ftp/pub/MarcosJ/03-CCM-lfi.pdf

  [Note: There are some differences in presenting and systematizing the field of LFIs in these two papers; the latter including also non-truth-functional semantics, tableaus and some ideas on quantificational LFIs.]
- As a monograph on LFI-Systems see (Marcos 2005).