Inconsistent Ontology

- An ontology of inconsistent objects is – in my eyes – the greatest challenge of/to paraconsistent mathematics and set theory.
- Given the strong paraconsistent program of true contradictions and a even mildly realistic theory of truth (containing in some – maybe even restricted – fashion the idea of correspondence), a true contradiction is *supposedly* made true by either an inconsistent fact (taking facts – at least for the moment – to be truth makers of statements) or by inconsistent objects. Like true contradictions they are just *there*.
- The challenge may not be that great for Naive Semantics given some mildly anti-realistic theory of truth (containing in some – maybe even restricted – fashion the idea that truth depends on justification), and observing that the inconsistent *objects* in that area are *sentences* only.
- The real problem are objects like the Russell set or the least inconsistent number. Isn't that too much to bear, even for a dialetheist?
- The most famous theory in the field is Meinong's ontology of possible, of inconsistent and of impossible objects, and their modes of being.
Inconsistent Ontology and Realism

- As a realist – even if you do not adhere to naive realism or extreme versions of metaphysical realism – you adhere to some principle that there correspond structured entities (facts or objects with properties) to true statements. The entities are – at least as much as our linguistic resources to describe them are partly sufficient – as the true statements say they are.
- This means that a truth like F(a) ∧ ¬F(a) means at first sight that the object a has property F and does not have property F. On second sight we worked with the idea of an extension of "F( )" and an anti-extension of "F( )", the extension being the set of entities fulfilling the criteria of F-ness, and thus being F; and the anti-extension being the set of entities fulfilling criteria of not being F, thus being not-F.
- For an anti-realist this may solve the problem of inconsistent objects, since being an inconsistent objects means nothing more for an anti-realist than that the objects fulfills inconsistent criteria. There is no claim on the anti-realist's side that there corresponds something to this in reality.
Inconsistent Ontology and Realism (II)

- The anti-realist can even explain how this may happen in case of ordinary objects: If predicates are employed to more or the less vague criteria or family resemblances to some prototype it may happen that one route of resemblance leads from the prototype of F to a, and another route leads via some intermediaries from a to a prototype of non-F. In the manner of weak paraconsistency one may argue that we have to be able to model theories that depict – at least implicitly – the world as containing inconsistent objects without ourselves to be committed to this picture. We need the formal tools (like LPQ) for this, but these tools themselves have no negative ontological impact. That is just like we need a logic to draw inferences in works of fiction (or about art) where some works are essentially inconsistent with respect to some object (e.g., some stories about time travelling or drawings by M. C. Escher).
- A realist cannot take this easy way out. For (most) realists properties are structures of objects – or parts or tropes... – and either you have them or not.
Inconsistent Ontology and Realism (III)

- In case of *sentences* – i.e. for a dialetheist view on naive semantics – the way out may be that a sentence is really an object that can have inconsistent properties without us having ontological scruples: A sentence being a dialetheia means that itself and its negation are provable. These are clear cut properties.
- What about the content of the dialetheia? If the Liar is really true there has to be some fact corresponding to this truth!
- The content of the semantic antinomies, once again, concerns facts *about language*. Given our mild form of realism that incorporates some idea that truth is also – besides aiming at correspondence – tied to justification we can accept inconsistent objects here, since this "merely" points to the inconsistent nature of our linguistic access to reality. That is a deep philosophical point – as dialetheism is – but it locates the inconsistent objects somewhere in the objects having the job of mediating between our mind and the rest of reality, these objects often being constituted by linguistic conventions.
Inconsistent Ontology and Realism (IV)

- The problem of inconsistent objects is much harder with respect to ordinary objects. If properties are structures of objects, and this means in the last analysis structures of distribution of matter and energy, then an inconsistent objects cannot exist, it seems, since either at some location there is matter or not.

- For a dialetheist the problem is naive set theory, given one is a realist about sets. A set, it seems, either is a member of another set or it is not, otherwise the including set could not be well-defined.
Meinong's Ontology

- The most promising place to turn to for an ontology of inconsistent objects is Alexius Meinong's *Gegenstandstheorie*. The logic of Meinong's ontology has to be, it seems, some kind of paraconsistent logic; and strong paraconsistent semantics may need something like Meinong's ontology. Thus the focus in this chapter is on Meinong's ontology and its formalisation by analytic philosophers.
- Meinong's theory has been ridiculed by analytic philosophers from Russell to Ryle. The idea of there being an object for any name or description (including the golden mountain or the round square) seems too strange.
- There has been a growing interest in Meinong's theory, however, with the advent of Free Logics (logics that allow for individual expressions that do not refer) and the intricacies of the semantic and logical analysis of context of belief. The logic of fictional objects also can be hardly referential. Semantics of modal logics talk about different modes of being (being actual, i.e. part of the actual world, vs. being merely possible) – so there may be a relationship to Meinong's ideas.
Meinong's Ontology (II)

- In Meinong's theory are – in sense of "are" to be explained" – all objects. There are also inconsistent and impossible objects.
- With inconsistent objects there seem to be inconsistencies, since the round square is round and is square, thus round and not-round.
- Meinong himself did not develop a logic to deal with inconsistent objects. He saw standard logic as fit for existing objects (these are the objects in space and time). Inconsistent objects are for him, because of their inconsistency, impossible objects (i.e. it is impossible that they exist [somewhere in space and time]).
- Meinong's basic idea is the rejection of any inherent connection between having properties (i.e. predication) and existence (i.e. metaphysics, in Meinong's use of the term). [This idea can be spelled out into several theses – to be explored shortly.]
- The point of attack is the Ontological Assumption saying (OA) Predication implies existence.
or the Referential Assumption saying (RA) Every singular term refers to some kind of being.
Noneism

- This attack on the Ontological and the Referential Assumption is generalised by Richard Routley (1979a) into noneism.
- Noneism is the theory of non-being. Meinong's more fundamental idea – behind his attack on (OA) and (RA) – is the claim that objects are beyond being. That idea is supposedly hard to grasp.
- Meinong does not claim that there are realms of being besides being in space and time (i.e. existence). There are no domains of possibilia (possible objects). And there are no "outer-domains" (like in some Free Logics) where even more strange objects like the round square reside.
- Meinong claims that objects are without being! The philosophical field that deals with objects in this generality is Gegenstandstheorie [Object Theory] (whereas metaphysics is concerned with existents only).
- "Noneism" is a name to Meinong's claim. Noneism claims, for example, that mathematics does not deal with anything having being; so there is no problem of abstract entities: numbers etc. are objects and nothing beyond that. The same goes for sets and properties (if taken as abstract entities). Noneism is the alternative to Platonism, it claims.
Noneism (II)

- Noneism goes beyond a *dual aspect theory* of meaning (that separates sense/meaning from reference), since it sees in such dual aspect theories only a version of (RA) in that these theories take (as in Frege) the sense sometimes as the referent of an expression.
- Any kind of shifting (like in Frege) *or* using intensions in intensional contexts (like in Carnap) is said to less adequate to our understanding of intensional contexts than a *straightforward* noneism such that
  
  (1) Peter believes Holmes to live in Bakerstreet.

  expresses a relation between Peter and (the object) Holmes.

- Objects can be *quantified*; there will be different kind of quantifiers though – some with existential impact, others not.
Postulates of Noneism

- Noneism can be characterised by a set of postulates:

  (P1) Everything is an object.

  (P2) Many objects have no way of being.

  (P3) Nonexistent objects have properties and are constituted in some way.

  (P4) Existence is no characteristic property of an object.

  (P5) Every object has an essence independent of its existence.

  (P6) Every object has its characterising properties.
Postulates of Noneism (II)

- These postulates have some immediate consequences:
  - by (P1) anything can be the object of belief, desire, fear – any propositional attitude – whether the object is possible or impossible.
  - by (P6) – also called the Characterisation Postulate – the golden mountain is golden, and the round square is round and is square.
  - by (P4) versions of the ontological proof of God's existence should be blocked.
  - by (P3) and (P5) the round square and the triangular square are different impossible objects.
  - by (P5) and the existence of impossibilia we can say:
    (2) There is something which necessarily does not exist.
    The first quantifier having no existential impact here.

- Postulate (P3) is called the Principle of Independence. As a universal claim it is the contrary to (OA).

- The Characterisation Postulate is a further claim to the well defined nature of nonexistents. It allows to draw modal conclusions (like (2)) from the definition of an object.
Noneism and Intensional Contexts

- As mentioned above, by (P1) anything can be the object of belief, desire, fear – any propositional attitude – whether the object is possible or impossible. The attitude is directed to the object itself, not to its representation or to the sense of the object's name.

- And given that we have quantifiers at our disposal that carry no ontological commitment we can freely quantify into intensional contexts:
  
  (3) Peter thinks of the round square.

  (4) (∃x)(Peter thinks of x)

  goes through.

- [Note that the failure of substitutivity in intensional contexts still obtains in noneism: If

  (5) The round square is Meinong's famous example for an impossible object.

  we still need not have that (6) is true if (3) is true, given Peter's ignorance:

  (6) Peter thinks of Meinong's famous example for an impossible object.]
Talking about Nonexistents

- The postulates have some immediate consequences as well for our way of using arbitrary individual expressions.
- We can recast the *Postulate of Independence* as a statement about sentences [cf. Routley 1979a, p. 14]:
  
  (P3') Sentences about singular objects have meaning independent from the existence or the possibility of these objects.

  (P3'') Many sentences about nonexistents have a truth value.

- Not every sentence about a nonexistent object has a truth value, since nonexistents are in most cases only partially characterised. We do not know how many rolls Holmes ate during his observation of the Baskervilles. (So a noneist logic may allow for truth value gaps.)

- [Note that (P3') and (P3'') are true in many Free Logics as well, cf. (Lambert 1991). The distinguished nature of noneism is due to postulates (P1), (P2), (P5) and (P6).]
Talking about Nonexistents (II)

- Assertaining which properties an existent has may be observable or be based on observation. Which properties a nonexistent has has to be determined differently.
- Whatever can be said with truth value (be it "true" or "false") about nonexistents can only be due to their definitions (as in the case of the round square) or due to the story told about them (as in the case of Holmes).
- So what we have here is basically a way to get sentences from other sentences. We do not access objects of any kind.
- Given that we already have the definitions or the stories before us, all these sentences about nonexistents have their truth value a priori.
Meinongian Logic

- The logic corresponding to Meinong's ontology cannot be a standard logic because of the presence of inconsistent objects.
- It cannot, as well, be a paraconsistent logic in which *Tertium Non Datur* holds, since Meinong allows not only for inconsistent, but also for incomplete objects (i.e. objects such that there is some property P such that neither P nor ¬P are true of them) – unless one introduces a distinction between sentence negation and predicate negation. Then *Tertium Non Datur* may hold for sentence negation (the non-application of the property making the negated sentence true), but not for predicate negation. Given two negations one needs to know what interrelations hold between these negations. Does, for example, ¬¬P(a)⇒P(a) hold? There has to be some *logic* for predicate negation if it is not just a tool of introducing additional predicates. Further on one may wonder whether natural languages have these two negations.
- If one kept a single negation the logic of Meinongian ontology might be some 4-valued logic like *BN4* [cf. Chap. 9] or *FDE* [cf. Chap 5].
Assessment of Noneism

- Dialetheism in semantics needs no special ontology of inconsistent objects if the inconsistencies are located within our linguistic frameworks. That a sentence can be shown to be true and can be shown to be not true points to the fact of inconsistent evaluations or derivations, but to no deep ontological mystery.
- Inconsistent theories in the sciences can be understood in the sense of weak paraconsistency, i.e. they may be modelled by LPQ-style quantificational semantics with inconsistent objects, but one does not believe that there really are these objects.
- You really need an ontology of inconsistent objects if you are a mathematical realist and your favourite mathematics is inconsistent, or if you are a dialetheist in a set theory, again taken realistically.
- You can have a dialetheist set theory with inconsistent objects if you are a noneist. Noneism in denying being to inconsistent objects is, however, to weak as foundation of paraconsistent set theory, since sets – unless you believe in noneism – are usually taken to exist (even if not in space in time). Noneism certainly is no version of realism.
Assessment of Noneism (II)

- Noneism is either a version of a substitutional understanding of individual expressions (such that we can use a non-referring expression and tell a story in which this expression occurs [for example a story about a unicorn]) in which case intensional contexts are to be understood in some way paratactically [in the tradition of Carnap's intensional isomorphy or Davidson's saying that],
  – or noneism is not comprehensible at all.
- The distinction between existents and possibilia is a clear one in modal semantics. Noneism claims even less being than possible being, but still uses the forms of "to be".
  – What an object beyond all being is supposed to be, is beyond me.
- The Independence Thesis seems at least in its noneist reading quite questionable. BeingRound in case of the non-being round square cannot be the same manner of being round like in the case of a penny coin, BeingGold in case of the non-being golden mountain cannot be the same manner of being golden (i.e. having some physical structure) like in the case of the gold bar. General terms seem to become ambiguous here!
What Do We Need "Items" for?

- The main argument that in any context we need an "item" to be talked about is not convincing. Given a dual aspect theory of meaning (i.e., reference vs. meaning in the narrow sense) some talk may have no referent, but still has meaning. Referentially it is about nothing, but we can rephrase the content of such talk. Giving this content does not presuppose an "item" that these sentences are "about".

- In Free Logic we are free to use any singular term we like. It just may turn out that it does not refer. This accounts for the free use of names and descriptions in ordinary discourse. We do not need the – further – assumption that everytime we use an expression that might refer to an object there "is" some object, although it might be non-existent and completely without being. Even allowing possibilia (or ersazist possibilia) that subsist without having physical existence may do a better job in a Free Logic framework than inventing non-being.

- So, if we are interested in inconsistent ontology this may be not because we have some use for inconsistent objects (apart from sentences) somewhere in philosophy, but because we are forced to consider ways to have inconsistent objects by our adherence to Naive Set Theory.
The Logic of Encoding Framework

- If one believes in abstract entities the problem of inconsistent objects seems to be less pressing, since anything may go on in that realm.
- A major alternative to noneism is Zalta's Logic of Encoding (LoE), which is an axiomatized ontology of abstract entities based on Second Order Modal Logic. LoE deals with several philosophical topics (like intentional contexts), but we look here just at its sub-theory of inconsistent objects and its way of interpreting/using Meinong's ideas.
- The central idea of LoE is the distinction between exemplification and encoding [following Meinong's pupil Ernst Mally].
- Exemplification is that way of ordinary predication in which a red pen is said to exemplify the property BeingRed. Encoding in distinction to this is the way abstract entities are determined. An abstract entity is the specific abstract entity it is by encoding some specific properties. The crucial point is that an abstract entity need not exemplify the properties it encodes.
- **Example:** The golden mountain encodes the properties of BeingGolden and BeingMountain (these encoding facts make up this specific abstract entity), but it need not exemplify these properties! An abstract entity has no more need to exemplify the properties it encodes, as a representation of a lake needs to be wet.
The Logic of Encoding Framework (II)

- The **LoE** framework thus denies the *Independence Thesis* (which now would cross illicitly from encoding to exemplification anyway).
- Several kinds of abstract entities exist (individuals, properties, relations, propositions).
- These abstract entities play all the roles *items* play in *noneism*.
- To have enough abstract entities around a principle of comprehension claiming that there is an encoding abstract entity to any property is postulated.
- To avoid paradoxes encoding is generally restricted to first order properties with a finite number of definitionally introduced exceptions.
- For Meinongianism this means:
  - there are abstract objects which encode contradictory properties
  - since these objects need not exemplify the properties they encode there are no true contradictions because of these objects
  - the underlying logic, thus, can be (extended) standard logic.
The Logic of Encoding

- To give you some idea of the formal LoE we may look at some of its ingredients:
- First of all it distinguishes exemplification formula \([F(a)]\) from encoding formula \([aF]\).
- If \(a\) encodes \(F\) then \(a\) necessarily encodes \(F\): \(\Diamond xF \supset \Box xF\)
- The logic uses quantified S5 modal logic (quantifying also over properties, where properties may be build by \(\lambda\)-abstraction) with identity and descriptions which designate rigidly and need not refer at all (using the usual precautions with respect to this dangerous combination).
- "\(E!(\cdot)\)" is used as an existence predicate with the intended meaning "existing in space-time".
- The ordinary objects are the existing objects. They are identical iff they necessarily exemplify the same properties:
  \[(\forall x,y)(O!(x) \land O!(y) \supset (x = y \equiv \Box (\forall F)(F(x) \equiv F(y))))\]
- Abstract objects are objects \(x\) that cannot exist in space-time: \(\neg \Diamond E!(x)\)
- Abstract objects are identical iff they encode the same properties:
  \[(\forall x,y)(A!(x) \land A!(y) \supset (x = y \equiv \Box (\forall F)(xF \equiv yF)))\]
- Two properties are identical iff they are encoded by the same objects.
The Logic of Encoding (II)

- The quantifiers range of single fixed domains of individuals or properties. Therefore the Barcan Formula holds: \((\forall x)\Box Fx \supset \Box(\forall x)Fx\)
- The essential axioms are principles of comprehension:
  
  \[(\exists x)(A!(x) \land (\forall F)(xF \equiv A))\]

  where A does not contain "x" free

So for every condition on properties (especially their combination by disjunction) there is an abstract object that encodes just these properties.

- Given this axiom one can prove that abstract description always refer:
  
  \[(\exists y)( y = \exists x(A!(x) \land (\forall F)(xF \equiv A)))\]

- For exemplification a comprehension principle can be derived by necessitation and existential generalization:
  
  \[(\exists F)\Box(\forall x)(F(x) \equiv A)\]

  where "F" is not free in A, which does not contain encoding formula or descriptions.

[The restrictions, which also apply to abstraction, are needed to prevent antinomies of encoding and comprehension.]
The Logic of Encoding – Assessment

One may assess the Logic of Encoding on several accounts.

Whether it is the best theory to deal with intensionality (its main concern) cannot be made out here, since this required extensive comparisons with rival approaches. Intensionality is not our main concern here, anyway.

Whether one should accept an ontology of abstract entities is another difficult question raising problems in epistemology and ontology (e.g. whether we need sets or classes in distinction to heaps or tropes …). This isn't our main concern here either. Given, however, that one allows for abstract entities one has the option to introduce a relation like encoding. And this may very well be a better way to make sense of Meinongianism than noneism.

Whether LoE is an alternative to dialetheism is a completely different topic. Zalta claims that his framework is an alternative. The point, however, is – again – that the framework can deal with some antinomies without yielding new ones only because its expressive power (e.g. with respect to encoding) has been artificially curtailed in a version of restricted comprehension. Furthermore, the theory does not contain its own meta-theory, which is expressed set-theoretically, as Zalta admits himself. This needs further study.
Inconsistent Ontology Again

- What have we seen so far?
- Given our philosophical agenda of integrating Naïve Semantics and Naïve Set Theory into our logical semantic framework we cannot settle for restricted means of expression (like the Logic of Encoding).
- Given the supposedly hard to swallow consequences of noneism it cannot be taken in its pure form either, it seems.
- Maybe – embracing abstract entities – there is a paraconsistent version of the Logic of Encoding which commits us only to its rather harmless pseudo-impossibilium while providing us with dialethic universality.
- Again [cf. the first slides in this chapter and Chap. 11] one might retract on the philosophical agenda: Universalism in semantics and epistemology I take to be far more important than sticking to Naïve Set Theory. One might be a dialetheist in semantics, but a proponent of some consistent sets + classes theory in axiomatic ontology, accepting some version of NBG set theory or Brady's theory of classes. The problem with the MYSTERY [cf. Chap. 2] may seem less troubling in the set/classes-case than in semantics and epistemology.
  – One shouldn't give up on Naïve Set Theory, however, so quickly.
Questions

• (Q1) Why can we say that the basic First Order principle of noneism is: F(∀xF(x))?

• (Q2) Noneism claims that mathematics is a "just so" story about objects that have no way of being. What are the fundamental epistemological and ontological problems of this view, especially in light of the usage of mathematics in the sciences?

• (Q3) In the Logic of Encoding the identity of abstract objects is not defined by having two candidates x, y exemplify the same properties, but by having x, y encode the same properties. Why is this crucial and unavoidable?
Exercises

- (Ex1) Prove in LoE: \( \neg (\exists x)(O(x) \land A(x)) \)

- (Ex2) Without the restrictions on \( \lambda \)-abstraction and comprehension one could assert the existence of an abstract object \( a \) that encodes the following property:

\[
(\exists x)(A!(x) \land (\forall F)(xF \equiv F = [\lambda x(\exists F)(xF \land \neg Fx)])
\]

Go with \( a \) through the cases of the ensuing antinomy!
Further Reading

- On the wisdom of the middleway in matters of realism between (extreme) metaphysical realism and idealism cf. (Bremer 1999a, 2000).
- Meinong's theory is presented in his Über Gegenstandstheorie (Leipzig, 1904 [in the volume Untersuchungen zur Gegenstandstheorie und Psychologie]) and Über Annahmen (Leipzig, 1902).
- Richard Routley, one of the founding fathers of paraconsistency and dialetheism, wrote a monumental monograph (1000 type scripted pages) Exploring Meinong's Jungle and Beyond (Routley 1979a) expousing not only his interpretation of Meinong's work, but also attacking the referential assumptions of standard semantics in his own right. Routley aims at a wider theory of noneism (and paraconsistency)
Further Reading (II)

- An exposition on the supposed logic of Meinong's theory is: Jaquette, Dale. Meinongian Logic. Berlin/New York, 1996. Jacquette does not use a paraconsistent logic, since he distinguishes between predicate and sentence negation. The inconsistent objects just have a property and its contrary, where predicate negation is not explosive. Jacquette also uses Meinong's distinction between nuclear properties, which are part of the essence of some object, and extra-nuclear properties, which are not allowed to enter into the definition of an object. But why should this be forbidden? The main reason to forbid this is to avoid the resulting antinomies, it seems.

- See also: Pasniczek, Jacek. "Paraconsistent vs. Meinongian Logic", in: (Carnielli/Coniglio/Ottaviano 2002), pp. 523-34.