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# Hypercontradictions

- The worst thing that could happen to a premise set in standard logic is that it becomes or turns out to be inconsistent. A contradiction (a sentence A, such that A and ¬A are given or even provable) is the worst case for standard logic. The set of consequences is thereby trivialized. Everything can be deduced, since *ex contradictione qoudlibet sequitur* holds.
- Such a simple contradiction is no problem for paraconsistent logics. To deal with such contradictions is the purpose of PLs.
- The question is whether there are sentences that are as desastrous to a paraconsistent logical system as contradictions are for a system of standard logic. One may first define such *hypercontradictions* and then see whether there are actually any. If there are provable hyper-contradictions a paraconsistent logic is as trivial as standard logic!
- Another problem may be the implausibility of the self-application of some of the dialetheist's basic statements. This may not be as bad as hypercontradictions, but disastrous for a (philosophical) presentation of the dialetheist's position.
- [We first look at the second problem, then at hypercontradictions.]

# Truth without Assertability?

- Strong paraconsistency (dialetheism) claims that some contradictions are true. Theories which entail contradictions might be correct. So some contradictions have to be true, since they are provable. Take, for example, the Liar in naive semantics

  (λ) is false.

  by familiar reasoning (valid in at least some paraconsistent logics) we arrive at the conclusions that (λ) is both true and false, its truth being derivable by *reductio* from the assumption of its being false.
- Now, something that is true should be assertible by any speaker towards an audience.
- Furthermore, being provable (λ) fulfills even the strictest condition that a semantics of assertability could insist on. The semantic battle between truth and assertibility does not apply to (λ).
- However, there might be another battle to be fought. I will explore some *pragmatic* constraints on assertability that might be strong enough to make (λ), although provable, not assertible.

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# Is Dialetheism a Dialetheia itself?

- What about the truth of "A is true" in case A is a dialetheia?
- In a many-valued logic it is possible that statements/sentences concerning the value of other statements/sentences are bivalent: "A is true" is true only if A is true; in case A is undecided "A is true" is simply false [cf. (Blau 1978, pp. 82-84)].
- What if A is dialetheia? Graham Priest claims that in this case "A is true" is a dialetheia itself [see (Priest 1979, pp. 238-40)!].
- The difficulty arises because of convention (T), which – neglecting paraconsistently the difference between object and meta language – we can state as:

$$\text{(T) A is true} \leftrightarrow \text{A}$$

- By Contraposition we arrive at:

$$\text{(T')} \neg\text{A} \leftrightarrow \neg(\text{A is true})$$

- If A is a dialetheia we have $v(\text{A},0)$ so by the definition of negation in **LP** we have $v(\neg\text{A},1)$. And with *Modus Ponens* applied to (T') we get:

$$v(\neg(\text{A is true}),1)$$

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# Is Dialetheism a Dialetheia itself? (II)

- and once again by the definition of negation:

  $v$(A is true,0)

- Furthermore it is true that A is true, since it is a dialetheia, therefore:

  $v$(A is true,0) *and* $v$(A is true,1) ↯

- "A is true" now is a dialetheia itself. We would have the truth table:

| A | A is true |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 0,1 | 0,1 |

  and similar for "A is false".

- This, however, would have paradoxical consequences. The main thesis of (strong) paraconsistency is that some contradictions are true. Now, this should have the form "(∃A)A is true" with respect to at least some contradiction/dialetheia "A". Since "A is true" is a dialetheia itself, the thesis of dialetheism will be a dialetheia.
- The main thesis of (strong) paraconsistency *would be antinomic*!
  [(Priest 1979) accepts this result – even feels happier about it (?). To me this consequence seems absurd.]

# Conditions on Assertability

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- A minimal condition on a philosophical thesis should be that it claims to be true *only*. Otherwise the triviality which should be avoided in case of theories containing contradictions would reappear.
- The *act of asserting* something can only fail with respect to a dialetheia, since the *felicity conditions of assertion* contain, at least:

  (i) that we claim something to be the case by citing reasons or giving a justification and

  (ii) see this information as worthy of being uttered, because the acceptance of this information would make a difference in the following discourse or action.

But in case of a dialetheia this only can misfire, it seems.

# Assertability, Option 1

- With respect to asserting dialetheias one may argue in the following way, which we consider as “Option 1”.
- An antinomy *asserts* nothing (in a pragmatic sense of "assertion" to be specified). There might be statements which *are* true and false at the same time, but there can be no *reason* to assert them in a theoretical debate (i.e, reasons besides training one´s vocal chords or being on stage etc.), since nothing is excluded by claiming them to be true. No possible state of the world (no piece of information) is rejected because of their assertion.
- Whereas belief is cognitive (accepting a dialetheia may lead to further beliefs) assertion is pragmatic and has to fulfil the further felicity conditions of assertion mentioned above.

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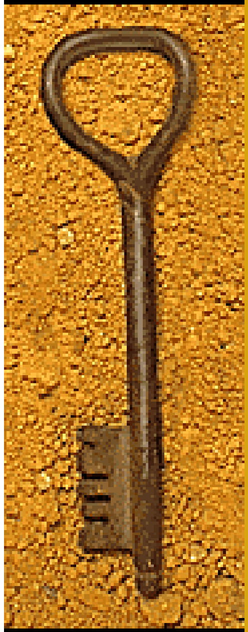

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# Assertability, Option 1 (II)

- The aim of assertion is truth, and *nothing but the truth*: only the "true only" excludes its opposite and, thereby, commits itself. In asserting something we commit ourselves against some opponent. Usually we do not consider this, since usually we do not argue in antinomic contexts. In non-antinomic context it is sufficient to show the falsity of the claim of the opponent to infer to the truth of our claim. In the case of a dialetheia we are not able to argue for it in this ordinary fashion, since in its case the success or the failure of our argumentation is irrelevant to justify the claim made. The dialetheia is validated in any case. Any justification given by us plays only in the hands of the opponent. Any argument that I will present to make the case of a dialetheia "A" will be an argument for "¬A" (i.e. the claim of my opponent).
- Therefore, we cannot assert contradictions/dialetheias with the *knowledge* of them being contradictory, since then asserting them is irrelevant or not preferable to the assertion of their negation.
- Contradictions – at least if we know them to be such things – violate the conversation maxim of *relevance*. We will be unable to *commit* ourselves.

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# Assertability, Option 1 (III)

- It will not help here to take the informational content of a sentence to be not those worlds or sentences it excludes but the information "it carries", i.e. the sentences it implies, since a dialetheia *by its very definition* and by the usual reasoning as applied, for example to the Liar, implies its own negation, and, therefore, implies everything its negation implies. A dialetheia and its negation have the same content. A dialetheia implies its own negation, and so, by transitivity, all that the negation implies, and vice versa: the share their set of consequences.
- Considering such pragmatic constraints asserting something is *more* than having a true belief accompanied by some justification.
- Because of these constraints asserting something fails in case of dialetheias or known contradictions. Notwithstanding that we have proved them we are not able to assert them. Therefore, dialethism as a philosophical position cannot be a dialetheia itself on pains of being not assertible, i.e. being no contender in the debate at all.

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# A Problem with LNC?

- Even the validity of the Law of Non-Contradiction (LNC) in some PLs does not commit the adherent of such a system to an antinomic thesis of dialetheism itself: from

  (1) $\models(\neg(A \wedge \neg A))$

  we get by the definition of validity the statement:

  (2) $\text{True}(\neg(A \wedge \neg A))$

  From this we get by the definition of negation:

  (3) $\text{False}(A \wedge \neg A)$

- The argument of the Aristotelian, who claims that there are *no* true contradictions, will continue:

  (4) $\neg\text{True}((A \wedge \neg A)$

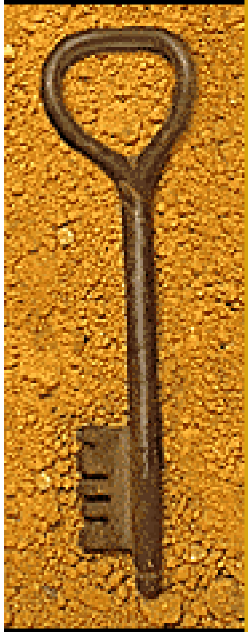
  and

  (5) $(\forall A)(\neg\text{True}(A \wedge \neg A))$

  since A has been chosen arbitrarily. So by the duality of quantifiers:

  (6) $\neg(\exists A)\text{True}(A \wedge \neg A)$

  which could be considered to be the thesis of the Aristotelian.



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# A Problem with LNC? (II)

- A dialetheist, however, is not committed to this conclusion, even if she accepts LNC.
- The step from (3) to (4) is not paraconsistently valid, since one cannot conclude paraconsistently from the *falsity* of a statement that it is not true (as well). That is the very issue of dialetheias.
- The negation of the thesis of the dialetheist (i.e., (6)), therefore, cannot be derived from the acceptance of the Law of Non-Contradiction.

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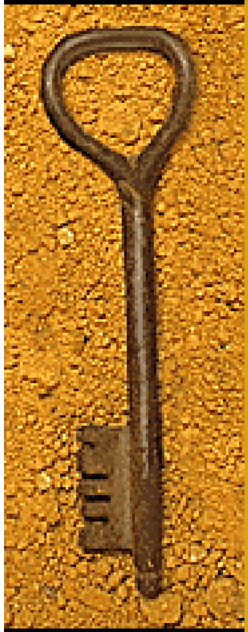
# An Assertability Principle

- How can we satisfy the constraints on assertibility and keep the thesis of dialetheism from becoming a dialetheia itself?
- The dialethist should say: Antinomies are true, but they are *not* assertible.
- Concerning an operator or predicate of assertion "ASSERT" it would *not* be valid:

  (*) ASSERT(True(A)) $\rightarrow$ ASSERT(A)
- Even if we can assert the truth of a dialetheia A (since, for example, we have a proof of A), this should not imply that we can assert A itself.
- The reason for this – given above – is that antinomies have no cognitive content.
- For metalogical reasons I might (should) be motivated to be a dialetheist and to assert that some contradictions are true, but I have no inclination to assert any antinomy simpliciter.
- I believe that the Liar is true, but I am not the Liar myself, I hope. What would I assert by it?



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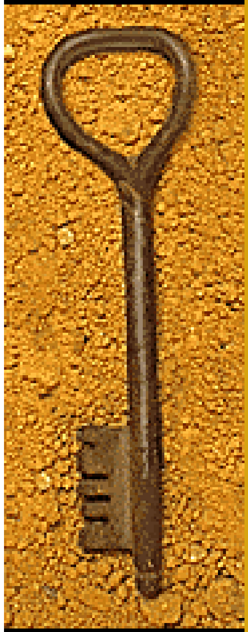
# Proof-Theoretic Adjustments

- The denial of the just mentioned principle or even stating its negation (that contradictions just cannot be asserted) could point to a pragmatic solution of the problem posed by Priest's "self-referential postscript".
- On the other hand we have the problem that our proof-theory and our semantics may result in the thesis of dialetheism being a dialetheia itself. If we *adopt* the principle that dialetheias should not be asserted that would make it unassertible. Since we want and have to assert it we have to adjust our proof theory.
- The argumentation so far saves us from asserting dialetheias which are, nevertheless, true. To *assert* dialetheism we have to ensure that dialetheism is no dialetheia *at all*. We have to block the derivation of it being antinomic.
- How could we block the derivation in if we consider convention (T) as beyond doubt? *Modus Ponens* seems even more beyond doubt. The truth conditions of negation cannot be altered without being deviant with respect to extensional operators (i.e., being deviant on too large a scale).
- The culprit has to be Contraposition.



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# Proof-Theoretic Adjustments (II)

- Without contraposition in convention (T) we could infer nothing from the falsehood of A (i.e. the truth of $\neg$A) concerning the truth value of "A is true". And this is how it should be in a paraconsistent semantics, since the truth values "true" and "false" should be independent enough from each other to allow for paraconsistent evaluations.
- In **LP** semantics Contraposition is valid as a consequence relation. The corresponding conditional is valid in **LP** anyway, since **LP** is an extension of classical propositional logic. It is difficult to see how to block the validity of Contraposition in **LP**, but because of its invalidation of *Modus Ponens* **LP**'s conditional "$\supset$" is doomed anyway as a representation of a real conditional. This means that the conditional in Convention (T) cannot be taken to be "$\supset$". Either **LP** has to be extended by another conditional invalidating Contraposition or **LP** cannot be the logic of dialetheism!
- As we have seen in Chap. 6 **SKP** does not have Contraposition as a rule of inference. The modal semantics of entailment can falsify it. Contraposition can be added to give us **SKP$^+$**, but that system then cannot be the logic of dialetheism.
  [As we have seen, as a consequence of dropping Contraposition indirect argumentation (e.g. Negation Introduction) is severely limited in **SKP**.]

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# Afterthoughts about the Liar

- Suppose you accept the proposal. It seems to work well with respect to a lot of antinomies. I might even assert that the Liar is true, since that is what we prove in naive semantics.
- But what about the corresponding claim that the Liar is false? If I claim that the Liar is a dialetheia I claim that it is true and false. By conjunction elimination I should arrive at

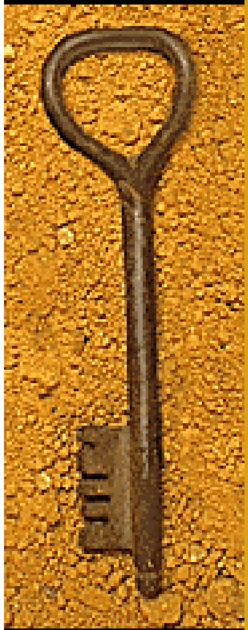
  (1) The Liar is false.

- Now, this sentence speaks about the Liar, and it says just what the Liar says, i.e.

  (2) $(1) \leftrightarrow (\lambda)$

- We seem to face a dilemma: either we assert a dialetheia itself, what we should not do, or if we block the procedure just outlined we face a problem of expressibility: Some semantic fact about the Liar (its being false) would become ineffable. And ineffability of semantic facts is unacceptable since it bereaves dialetheism of its main argument against hierarchy conceptions of semantics which are ineffable themselves.

# Afterthoughts about the Liar (II)



- Two ad hoc solutions could be: either to weaken the ban on asserting dialetheias (allowing some exceptions to a default rule) or to invent some other description of the Liar.
- If the Liar is the dialetheia paraconsistent logicians are most concerned with I could assert
- (3) "The dialetheia that paraconsistent logicians are most concerned with" is false.
- If we treat "is false" as a predicate we are still allowed to substitute different *names* of an expression for each other (i.e. substitute (λ) for "the dialetheia that paraconsistent logicians are most concerned with"), but if we now assert of (3) that it is true or that it is false, i.e.
- (4) "The dialetheia that paraconsistent logicians are most concerned with is false" is true.

  we have *inside* the scope of the truth predicate a name of (3) and into this we cannot substitute (λ) for "the dialetheia that paraconsistent logicians are most concerned with" (this being substitution into quotation marks).
- This blocks: (*) (4) ↔ (λ)

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# Afterthoughts about the Liar (III)

- So we can assert that it is true that the Liar is false and, therefore, that the Liar is false, i.e. we could assert our semantic fact concerning the falsity of the Liar.
- So some assertions about dialetheias are dialetheias themselves, like (1) is. Some are not, like (4).
- And that is all we need for consistently asserting the thesis of dialethism. If only we can express the thesis of dialetheism in some way – and in some way with respect to any single dialetheia like the Liar – Priest´s problem of self-application is solved.
- Since we already have seen that the thesis of the Aristotelian cannot be proven, we are saved.
- Surely more needs to be said about this.

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# Assertability, Option 2

- "Option 1" on the assertability problem has it that dialetheias are not assertible. Even if they aren't this poses no special problem for dialetheism.
- But, maybe, they are! In arguing for dialetheism it has been asserted on these very slides that some specific contradiction (like the Liar) is true – hasn't it?
- Why shouldn't the reasons that make one believe that some contradictions are true also be exploited to make one assert rationally that the Liar is true? This idea will be considered as "Option 2".
- The background of this alternative is to exploit the idea of bivalent truth operators within a suitable paraconsistent logic [cf. Chap. 4].
- "$\Delta$A" says that A is true only, "$\nabla$A" that A is false only, "°A" says that A is consistent "•A" says that A is contradictory. We can then say – and these being *just* true – that the Liar is true, false, not simply true, not consistent, and so on.
- Thus dialetheism can fulfil the traditional condition on any decent theory: that it claims to be just true (and not only as true as its negation).

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# Assertability, Option 2 (II)

- Dialetheism is thus no form of trivialism (that everything is true). The trivialist proposes $(\forall A)(TA \wedge T\neg A)$ or $(\forall A)(TA \wedge FA)$. The dialetheist claims $(\exists A)(TA \wedge FA)$, but also $(\exists A)(TA \wedge \neg T\neg A)$, and $(\exists A)\nabla A$.
- And given some formal system some formulas can be exhibited having these properties (e.g., defining a bottom particle $\perp$ with $\nabla\perp$ being valid). $\top$ can be defined as the top particle with $T(A \vee \neg A)$, being true only. The bottom particle $\perp$ can be defined as $\nabla(A \vee \neg A)$, being false only. Note that – in contrast to even the intuitionist negation rules – $\perp \equiv (A \wedge \neg A)$ need not hold if $\alpha$ is a dialetheia, since then $T(A \wedge \neg A)$ and $\nabla$ is incompatible with T.
- To have and use the (T)-scheme at the same time as these operators (be it for the operator "T" or "Δ") we need some revisions in the logic of the conditional, like giving up on the unrestricted validity of Contraposition. "Option 1" and "Option 2" don't argue about that.
- $TA \wedge \nabla A$ is a well-formed formula, but false only. The language of this version of dialetheism thus contains formula that can be evaluated only as being simply false [in contrast, say, to **LP**]. These formulas, of course, cannot be derived

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# Assertability, Option 2 (III)

- Consider the Liar again: Saying $T\lambda$ is thus simply true: $\Delta T\lambda$. This does not exclude that $F\lambda$ is also simply true: $\Delta F\lambda$. Now it *seems* that saying of the Liar that the Liar is false is just what the Liar is saying
- (i) $F\lambda \equiv \lambda$

  Then we might have
- (ii) $FF\lambda$

  and this contradicts $\Delta F\lambda$! But to derive (ii) we use either
- (iii) $F\lambda = \lambda$

  taking the sentences as objects and expressing their identity, or
- (iv) $\vdash (F\lambda \equiv \lambda)$

  which may be a *petitio* in the argument under consideration, and then substitution of identicals or substitution of equivalents. The equivalence thesis (iv) may be wrong.
- And substitution of identicals is one of those inferences restricted to consistent objects (to which $\lambda$ does not belong) [cf. Chap 7 and 20]. Even if (iv) is not wrong deriving $FF\lambda$ supposedly has to use some form of detachment, which again is restricted to consistent sentences (to which $\lambda$ does not belong).

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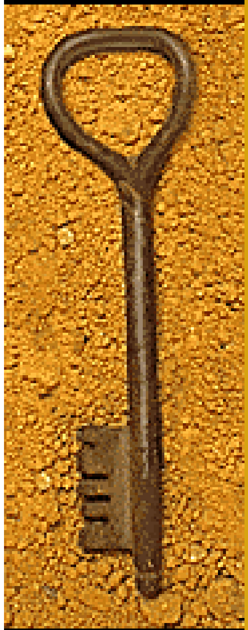

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# Assertability, Option 2 (IV)

- There is now a simple way to assert contradictions: Let us take it that F$\lambda$ can be believed and – being bivalent – can be asserted. Asserting T$\lambda$ or F$\lambda$ certainly fulfils some purpose, be it in explaining dialetheism or in arguing with opponents of dialetheism. What about $\lambda$ itself? What could be the purpose of asserting $\lambda$ when one could assert $\neg\lambda$ as well? Can asserting an antinomic sentence have any purpose at all?
- Given that the dialetheist is engaged in discussions about dialetheism it may be important to affirm her position by giving an example of what is a true contradiction. This can be done by affirming the antinomy itself, since we and the dialetheist take assertion to involve being convinced of the affirmed sentence being true (being *at least* true in the dialetheist's case). So if asserting A can be taken as asserting TA (not necessarily $\Delta$A in the dialetheist's case) and T$\lambda$ may be useful in a discussion about dialetheism, asserting $\lambda$ has its place as well.
- In memory of the distinction between object- and meta-language, dropped by the dialetheist, one may call this a *meta-assertion* of an antinomy. So there are occasions on which it is rational for a dialetheist to assert a contradiction, like I did on these slides.
- Are there – apart from the just given purpose of uttering $\lambda$ as a hidden/implied utterance of T$\lambda$ – other affirmative uses of $\lambda$?

# Assertability, Option 2 (V)

- There may be some metaphysics that helps. Negative facts have had a bad press in metaphysics. Again, however, it seems that a *general* commitment to negative facts is as superfluous as a general acceptance of any contradiction being true. The dialetheist has to accept only very special negative facts. In the case of Liar-like antinomies these facts consist in the negation of a sentence being as provable as the sentence itself. There is no further fact "behind" this. Since the proof is an existent *something* one may even speak of a positive fact here, like the intuitionist bases the claim for $\neg A$ not on the absence of reasons for A, but on the (positive) proof of $\perp$ from premise A [cf. Priest 1987: 87].
- The case is different with the set-theoretical antinomies, since here we seem to have the negative fact of the Russell set not belonging to itself besides the (positive) fact of the Russell set belonging to itself. These negative facts – one may argue – have their residue in the realm of abstract objects, however; much may be going on there. (If one takes sets not as abstract entities dialetheism in set theory may be a problem for realists [cf. Chap. 17].)

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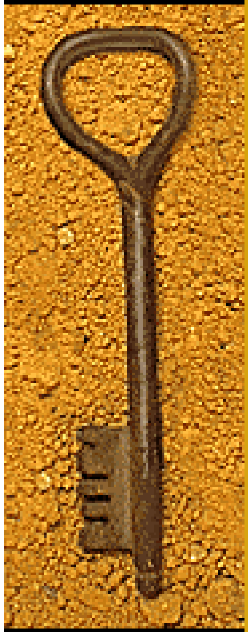
# Assertability, Option 2 (VI)

- Thus, if A entails ¬A and vice versa, and both are of interest in as much as the fact corresponding to ¬A is not just the *absence* of the fact corresponding to A (as an "ordinary" supervenient negative fact would be) substantial metaphysical assumptions come to light:
- I. Both facts are substantial (and interesting), and it is a further substantial *metaphysical* fact that although they do not stand to each other like contradictory sentences do in **PC**, not both can be false only [corresponding to the theorem $\vdash(\neg(\nabla A \wedge \nabla\neg A))$].
- II. The explication given above (the negative fact being the provability of ¬A) seems metaphysically questionable then, since why should it not be possible that we do *not* have proofs of either of them. Expressed in terms of truth (and *tertium non datur*) the option of both being only false can be excluded, but leaving aside the *provability* of ¬A what should be the negative fact corresponding to ¬A? One might settle for a metaphysical *tertium non datur* and simple negative facts (like the Liar *being* false, provably so or not).
- Giving up *tertium non datur* (in logic or metaphysics) is no real option, since the argument can be repeated with Strengthened Liars for multi-valued or gap semantics.



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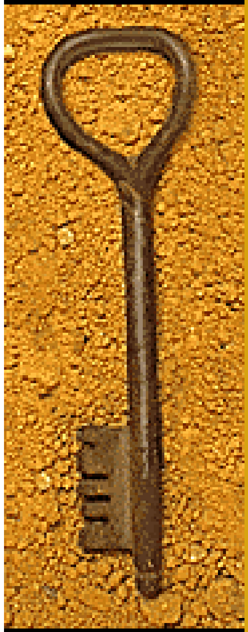

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# Assertability, Option 2 (VII)

- If there is a known metaphysical principle that excludes that both A and ¬A are only false, asserting one of them has no point in the sense of rejecting some modally available/accessible/possible fact. The same is true, however, with respect to any logical truth! They do not exclude anything either. Asserting dialetheias *simpliciter* (i.e., without semantic operators) thus has the same merit or futility like asserting logical laws *simpliciter* .
- The second alternative on the assertability problem may thus provide a more comprehensive approach in which we see dialetheism as being true only on the one hand, and see the rationality of asserting the dialetheias themselves straight forwardly, if only we accept some metaphysics of negative facts.
- Further on we may now have more elements for a dialetheic distinction between assertion, rejection and denial.

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# Assertion, Rejection, Denial

- Given that there is independent ground for $\neg A$, accepting $\neg A$ does not exclude accepting A. In contexts we know to be consistent we may reason to $\neg A$ without independent grounds on the basis of $\neg TA$ and *tertium non datur* (or some version of this disjunctive syllogism like reasoning). Since in case of antinomies accepting $\neg A$ does not exclude accepting A, accepting $\neg A$ should not be the same as rejecting A.
- Rejecting A cannot be understood by a dialetheist as affirming $\nabla A$. Rejecting A would thus be incompatible with affirming A (i.e. affirming TA). One needs a distinction then between affirming $\nabla A$ and affirming $FA[T\neg A]$. Sticking with the usage – arguably – of standard logic let us take affirming FA as *rejection* and affirming $\nabla A$ as *denial* of A [DENY(A)].
- Whereas there are situations in which a dialetheist accepts both A and $\neg A$, there are no situations in which a dialetheist accepts and denies A at the same time. As the foregoing distinction shows there is one kind of contradiction that (even) a dialetheist cannot support:

  $$\neg(\text{ASSERT}(A) \wedge \text{DENY}(A))$$

  since TA and $\nabla A$ are semantically incompatible.

# Assertion, Rejection, Denial (II)

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- Another simple point is that no-one (including the dialetheist) can have pragmatic contradictions: Speech acts being bodily movements that either occur or do not, there is not pragmatic parallel to having it both ways, i.e.
- $\neg(\mathrm{ASSERT}(A) \wedge \neg\mathrm{ASSERT}(A))$
- This instance of the accepted tautology $\neg(A \wedge \neg A)$ expresses not only a semantic exclusion the dialetheist accepts (and sometimes nevertheless supersedes), but the absence of the mysterious feat of asserting something and not doing it at the same time.
- There is no pragmatic dialetheism (without a verbal manoeuvre of redefining “not asserting” on the line of “asserting $\neg A$).

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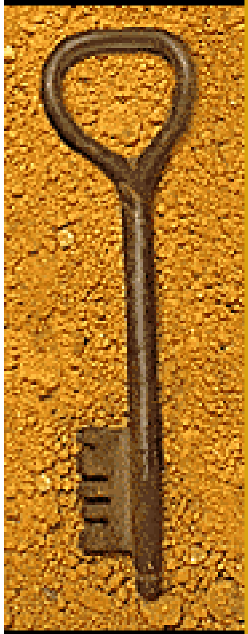
# Hypercontradictions Defined

- Classical logic system become trivial because of paradoxes like the Liar and rules of inference like *ex contradictione qoudlibet*. Dealing with the Liar a paraconsistent system (by revision of some logical principles) blocks spreading triviality.
- A *hypercontradiction* is a paradox that can arise within this new paraconsistent system and threatens it with triviality (e.g. by proving "1 = 0", so that *all* statements will be both true and false, taking the numbers to represent truth values in interpretations *v*).
- The philosophical merits of dialetheism (especially being able to have a semantically closed language) are endangered by hypercontradictions, since they would make this theory trivial.

- [Note: To have an exact concept of x does imply neither that there is an x nor that one would like to have an x around.]



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# The Everett Case

- Everett was one of the first to come up with hypercontradictions. They are a semantic version of the Strengthened Liar:

  ($\lambda$')                    ($\lambda$') is not true

- The Strengthened Liar ($\lambda$') is still mapped to {0,1}, arguing by cases for some interpretation function $v$:
  Suppose $0 \in v(\lambda')$, then ($\lambda$') is true, so $1 \in v(\lambda')$, so $v(\lambda') = \{0,1\}$, and if $1 \in v(\lambda')$ then accordingly.
- Given the four evaluations we can have in a **LP** style semantics (i.e. Truth, Falsity, Simple Truth, Simple Falsity) another Liar is:

  ($\lambda$'')                    ($\lambda$'') is simply false.

  This sentence takes into account that some sentences are both true and false, and claims of itself to be false *only* ("simply false").
- Syntactically it just leads to the contradiction ($\lambda$'') $\wedge \neg$ ($\lambda$'') within a suitable paraconsistent system, but from a semantic point of view it leads into disaster:

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# The Everett Case (II)

- Suppose $(\lambda'')$ is true, then the set of truth values $(\lambda'')$ is mapped to are either $\{1\}$ or $\{0,1\}$. But, on the other hand, if $(\lambda'')$ is true, by convention (T) $(\lambda'')$ obtains, so that the set of truth values $(\lambda'')$ is mapped to is $\{0\}$. So, if $(\lambda'')$ is true $\{0\} = \{1\}$ or $\{0\} = \{0,1\}$, which gives by extensionality of sets that $1 = 0$, i.e. triviality!
- So we end with $1 = 0$, which is the end of all distinctions between true and false sentences in the semantics given so far.

# Evaluation Relations

- To avoid this case we switch to an evaluation relation [as we have already done in previous chapters]:

  $$\neg(v(A,1) \Rightarrow \neg v(A,0)) \wedge \neg(v(A,0) \Rightarrow \neg v(A,1))$$

- Truth and validity are defined then:

  (D1) A is true in $M_i := v_i(A,1)$

  (D2) A is valid $:= (\forall v)v(A,1)$

- Considering now (λ’’) we have

  $$(\lambda'')\quad (v((\lambda''),0) \wedge \neg v((\lambda''),1))$$

- Take (λ’’) to be true, i.e. $v((\lambda''),1)$, then $v((\lambda''),0)$. Take (λ’’) to be false, i.e. $v((\lambda''),0)$, then $\neg v((\lambda''),0)$ or $\neg\neg v((\lambda''),1)$. By a property of negation we have that $\neg v((\lambda''),0)$ implies $v((\lambda''),1)$, since we have no truth value gaps here. So $v((\lambda''),1)$ implies $v((\lambda''),0)$, and vice versa. This comes, at most, to:

  $$v((\lambda''),1) \;\; \textit{and} \;\; v((\lambda''),0),$$

- (λ’’) being a dialetheia as it should be.

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# Truth Operators and New Antinomies

- The truth operators introduced into **LP** [in Chap 4] and to be used later [in Chap. 20] were not intended to be some kind of many-valued solution to the antinomies.
- That they give birth to further antinomies was to be expected. Consider the two following versions of a supposed Strengthened Liar:

$$(\lambda+) \qquad \neg\Delta(\lambda+)$$
$$(\lambda{+}{+}) \qquad \nabla(\lambda{+}{+})$$

- Arguing by the four cases (of truth operator assignments) one derives at

$$(+) \qquad F(\lambda+) \equiv \Delta(\lambda+)$$
$$(++) \qquad \Delta(\lambda{+}{+}) \equiv \nabla(\lambda{+}{+})$$

- But these are "merely" equivalent, given an evaluation *relation v* to:

$$(+)' \qquad v((\lambda+),0) \equiv \neg v((\lambda+),0)$$
$$(++)' \qquad \neg\, v((\lambda{+}{+}),1) \equiv \neg v((\lambda{+}{+}),0)$$

- (++)' is no antinomy at all. (+)' is our old Liar in new clothes, but so it is not worse than the old Liar.

# No New Strengthened Liars

- It should thus already be mentioned that these systems of paraconsistent logic hinted at above have sentences that *look* like Strengthened Liars (e.g. sentences saying of themselves that they are *false only*). Switching to evaluation relations and the restrictions on rules in proofs, however, avoids getting hyper-contradictions. The only interesting observation with respect to (some of) these Strengthened Liars is that they seem to be incapable of achieving what they assert of themselves (i.e. being false only).

- That even evaluation relations do not suffice and that such antinomies like (+)' are worse is the claim behind Bromand's hypercontradiction.

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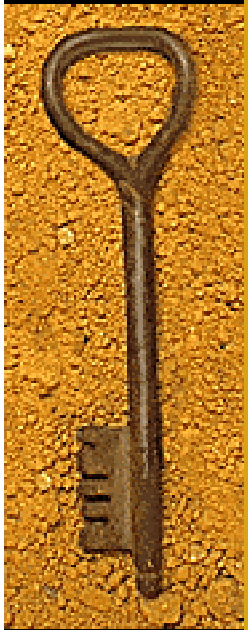
# Bromand's Hypercontradiction

- Joachim Bromand developed two versions of a hypercontradiction that is supposed to work *even* for a semantics using an evaluation relation!
- One of them can be presented in the form of this deduction:

| | | |
|---|---|---|
| 1. | $(\forall A,v)(\exists x)(x=\{y \mid v(A,y)\})$ | (Naïve) Comprehension |
| 2. | $d(A)=\{y \mid v(A,y)\}$ | function picking out the set of truth values given some formula A |
| 3. | $(\lambda''') \Leftrightarrow (d(\lambda''')=\{0\})$ | Definition of some Liar sentence, |
| 4. | $(\forall A)(d(A)=\{0\} \vee d(A)=\{1\} \vee d(A)=\{0,1\})$ | Semantic principle |
| 5. | $d(\lambda''')=\{0\} \vee d(\lambda''')=\{1\} \vee d(\lambda''')=\{0,1\}$ | Universal Instantiation, 4 |
| 6. | $(\forall A)v(A,1) \Leftrightarrow A$ | Convention (T) |
| 7. | $v((\lambda'''),1) \Leftrightarrow (\lambda''')$ | Universal Instantiation, 6 |
| 8. | $d(\lambda''')=\{0\}$ | Assumption |
| 9. | $(\lambda''')$ | Substitution of Equivalents, 3, 8 |
| 10. | $v((\lambda'''),1)$ | Detachment, 7, 8 |
| 11. | $d(\lambda''')=\{y \mid v((\lambda'''),y)\}$ | d-function for $(\lambda''')$ |



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# Bromand's Hypercontradiction (II)

| | |
|---|---|
| 12. $1 \in d(\lambda''')$ | Definition of d, 11, 10 |
| 13. $1 \in \{0\}$ | Transitivity of "=", 8, 12 |
| 14. $1=0$ | Extensionality of sets, 13 |
| 15. $d(\lambda''')=\{1\}$ | Assumption |
| 16. $1 \in d(\lambda''')$ | Definition of d, 15 |
| 17. $v((\lambda'''),1)$ | Definition of d, 16 |
| 18. $(\lambda''')$ | Detachment, 7, 17 |
| 19. $d(\lambda''')=\{0\}$ | Substitution of Equivalents, 3, 18 |
| 20. $1 \in \{0\}$ | Transitivity of "=", 16, 19 |
| 21. $1=0$ | Extensionality of sets, 20 |
| 22. $d(\lambda''')=\{1,0\}$ | Assumption |
| … 28. $1 = 0$ | see (16)-(21) |
| 29. $1=0$ | Dilemma, 5, $\underline{8}$, $\underline{15}$, $\underline{21}$ |

- Line (29) states the needed result, which can be then employed thus:

# Applying Bromand's Hypercontradiction

...

| | |
|---|---|
| 30. $v(\mathrm{C},1) \vee v(\mathrm{C},0)$ | Semantic principle ("no gaps") |
| 31. $v(\mathrm{C},1)$ | Assumption |
| 32. $v(\mathrm{C},0)$ | Substitution of Identicals, 29, 31 |
| 33. $v(\mathrm{C},0)$ | Assumption |
| 34. $v(\mathrm{C},1)$ | Substitution of Identicals, 29, 33 |
| 35. $v(\mathrm{C},1) \wedge v(\mathrm{C},0)$ | $\wedge$ Introduction, 32, 34 |
| 36. $(\forall \mathrm{C})(v(\mathrm{C},1) \wedge v(\mathrm{C},0))$ | Universal Generalisation, 35 ■ |

– That is triviality!

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# Getting Back on My Rocker

- Does the presence of a hypercontradiction bring us back to the classical solution of the paradoxes?
- That is not so. For the simple reason that there is no solution to the paradoxes in classical logic. The hierarchy conception says we are always talking from some level in the hierarchy, and *at the same time* makes a general statement to the effect that constructing the semantics of a language (level) we just go one level up. – This I called the MYSTERY [see Chap. 2].
- So it is instructive that Brendel (1992) after "rejecting" paraconsistency starts talking about Wittgenstein’s ladder or that which cannot be expressed at all, Bromand (2001) sums up his investigation that it "can be" that his thesis of inexpressibility (of the hierarchy, respectively of truth) cannot be expressed. That is no "can be", it just *cannot* be expressed, if true.
- Dialetheism avoids MYSTERY at the price of accepting some true contradictions.

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# Inconsistent Numbers to the Rescue

- Inconsistent arithmetic can have finite models. In a finite model there is a largest number. One might suppose that the largest number is a *vastly* large number beyond these limits. Therefore we do not know what this number is, but – realistically speaking – it is there. Let us call it "m". So for this number we have m = m+1, since there is no larger number. In fact there are no numbers m+1, m+2…, we only express the fact that there are no such numbers by m = m+1 = m+2 … [See Chap. 12 and (Priest 1994)].
- The number m (respectively the numeral "m") has to be an exception to the rule of substitution of identicals, otherwise one could reason to a=b for all numbers (by principles valid even in inconsistent mathematics)

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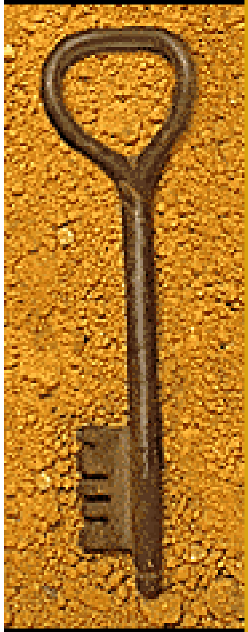

# Restricted Functionality

- In Chap. 5 on **SKP** we said:
  In case of Substitution of Identicals we introduce the set $\Xi$ of inconsistent objects (i.e. objects such that: $F(a) \wedge \neg F(a)$ for some F).
- Let $\delta$ be the denotation function of quantifica-tional semantics. Then:

$$(R13)\ \delta(á) \notin \Xi\ \&\ \Gamma \Vdash á=é\ \&\ \Pi \Vdash P(é) \Rightarrow \Pi \cup \Gamma \Vdash P(á)$$

If á does not denote an inconsistent object then substitution applies.

- We could also modify the rule (R13), using the least inconsistent number m and a denotation function *d( )*:

$$(R13')\quad d(á) \neq m\ \&\ \prod \Vdash á=é\ \&\ \Gamma \Vdash P(é) \rightarrow \prod \cup \Gamma \Vdash P(á)$$

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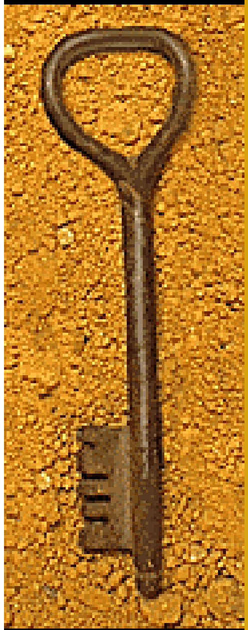
# The Bold Claim

- A sufficiently rich semantically closed language has to contain inconsistent arithmetic, so it contains m.
- Which number we take to represent the truth value "true" is conventional, so let us take m instead of 1.
- If we do not take "*v*(C,1)" as atomic – although it certainly looks that way from a syntactic point of view –, we may just give up transparency. If we take "*v*(C,1)" to be atomic, we have to give up functionality as well.
- Then lines 30-35 of applying Bromand`s hypercontradiction will not go through! Even if 0 = m, we cannot derive disaster. Triviality does not ensue.
-  – Can one keep a straight face proposing this? Is this an ***ad hoc*** solution or an *ad hoc* **solution**?
- Another solution – not committed to the inconsistent number m – would be the logic of (weak) identity that some extension of **LP** offers [see Chap. 4].



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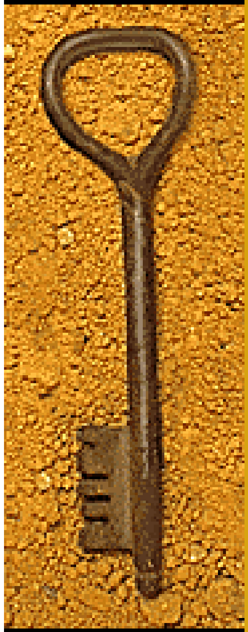

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# Other Problems with Functionality

- Starting from the work of Peter Geach there has been established a field called "Relative Identity (Logic)". The basic idea is that it makes no sense to speak of absolute identity (at least not in all contexts and concerning all questions of identity). Identity is (at least sometimes) relative to being the same x of a kind *F*. Van Inwagen (1995) develops some of these ideas in a little logical system of relative identity. Symmetry and Transitivity of relative identity hold, but substitution of identicals *fails*, since otherwise the absolute concept of identity could be regained. (Absolute identities are *substantial* assumptions then.)
- Pardey (1994) argues that in ordinary language contexts R(a,a) cannot be seen as substituting into R(a,b) in case that a=b. The idea that R(a,a) is a special case of R(a,b) he calls the "reflexivity thesis" in distinction to which he claims that in ordinary R(a,b) statements it is *presupposed* that a ≠ b. A presupposition cannot just be made a part of the sentence without producing strange sentences like "He reminded all of his audience, except himself, that …" Given this it is not valid in general that transparency holds, "a = b" is rather used to cancel the presupposition of non-identity made with "R(a,b)"

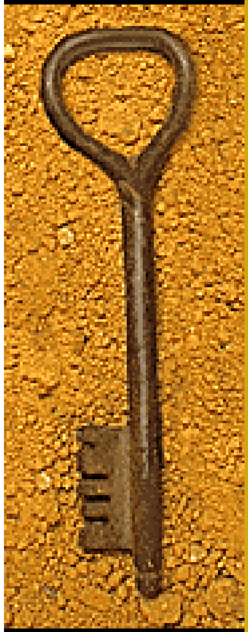
# Other Problems with Functionality (II)

- Krause and Beziau (1997) discuss several versions of stating Leibniz' Law and claim that quantum objects fail to meet it, since they are identical within their group although being different objects. They discuss some logics that violate identity in the form that the so called identity law for the conditional

  $$A \vdash A$$

  is not valid in them.

  The deviant behaviour of the quantum objects is, however, not mainly a problem of functionality (since quantum objects of some group indeed inherit their properties fulfilling functionality), but with identity in general; they demand something *stronger* than Leibniz' Law.



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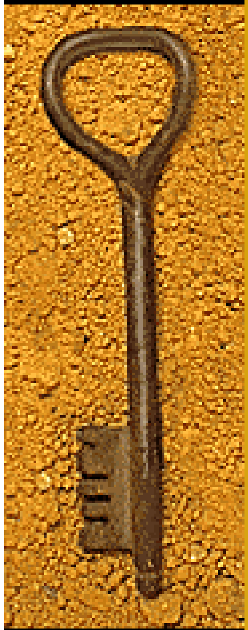

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# Assessment

- The implausibility of asserting contradictions and of the philosophical positions held by oneself to be true and false at the same time can be avoided given a further restriction of the preferred paraconsistent logical system (here: dropping Contraposition).
- Dialetheism is/remains a clearly statable theoretical position.
- The danger of the known hypercontradictions can be banned. This again requires a restriction of paraconsistent logic, in this case with respect to the logic of identity. This solution is not optimal, since it works either with the idea of inconsistent objects, which is problematic itself [see Chap. 17], or with a notion of identity (in **LPQ**$^{=}$) that is very weak. Dialetheist's should go on looking for a better solution to this problem. There has to be one, since there is no going back to MYSTERY.
- Tomorrow somebody may come around with a new supposed hypercontradiction. So this problem remains virulent. The same is true, however, for standard set theory or many other interesting theories.

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# Questions

- (Q1) Priest sometimes defends dialetheism by denying that a rule of reasoning just employed by his critics is not valid in "the paraconsistent logic". How could Priest react to the argument that a dialetheia and its negation share their information content, since by transitivity the dialetheia implying its negation implies what the negation implies?
  What can be said against this defence in turn?
- (Q2) Someone may say that to avoid giving up Contraposition in general the conditional in Convention (T) should be *sui generis*. What is the problem with that proposal?

# Exercises

- (Ex1) Show by tableaus that Contraposition as a consequence relationship holds in **LP**. [see Chap. 4]
- (Ex2) Go through the argument assuming ($\lambda$'') false.



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# Further Reading

- For the problem of self-application see: (Priest 1979, Postscript). “Option 1” here uses (Bremer 1999), “Option 2” – on the contrary – uses (Bremer 200x). On assertibility conditions see (Sainsbury 1988, Chap. 6).
- On the Everett hypercontradictions see: Everett, Anthony. "A Note on Priest´s 'Hypercontradictions'", *Logique et Analyse*, 1993, pp. 39-43; "Absorbing Dialetheia?", *Mind*, 103, 1994, pp. 413-19; "A Dilemma for Priest`s Dialethism", *Australasian Journal of Philosophy*, 74, 1996, pp. 657-67.
- On Bromand's hypercontradictions see: Bromand, Joachim. *Philosophie der semantischen Paradoxien*. Paderborn (Mentis), 2001; "Why Paraconsistent Logic Can Only Tell Half the Truth", *Mind*, 111, 2002, pp.741-50.
- The part on hypercontradictions uses (Bremer 200x).
- On the involved inconsistent mathematics see Chap. 12.