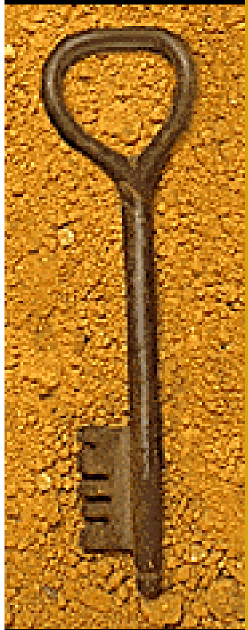
Deontic Logic and Meta-Ethics



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- Deontic Logic as been a field in which quite apart from the questions of antinomies "paradoxes" have played a decisive roles, since the field has been invented. These paradoxes (like *Ross' Paradox* or the *Good Samaritian*) are said to show that some intuitively valid logical principles cannot be true in deontic logic, even if the same principles (like deductive closure within a modality) are valid in alethic modal logic. Paraconsistency is not concerned with solutions to these paradoxes.
- There are, however, lots of problems with *ex contradictione qoudlibet* in deontic logic, since systems of norms and regulations, even more so if they are historically grown, have a tendency to contain conflicting requirements. To restrict the ensuing problems at least a weak paraconsistent logic is needed.
- There may be, further on, real antinomies of deontic logic and meta-ethics (understood comprehensively as including also some basic decision and game theory).

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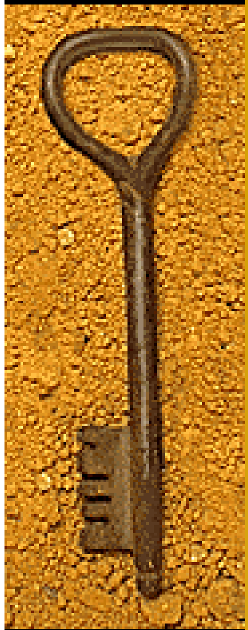
Inconsistent Obligations

- It often may happen that one is confronted with inconsistent obligations.
- Consider for example the rule that term papers have to be handed in at the institute's office one week before the session in question combined with the rule that one must not disturb the secretary at the institute's office if she is preparing an institute meeting. What if the date of handing in your paper falls on a day when an institute meeting is prepared?
  Entering the office is now *obligatory* (i.e. it is obligatory that it is the case that N.N. enters the office, $O(p)$), and at the same time it is *forbidden* (i.e. it is obligatory that it is not the case that N.N. enters the office, $O(\neg p)$).
- Now, in standard Deontic Logic we have a conjunction principle

  (1) $\vdash O(p \wedge q) \equiv O(p) \wedge O(q)$

  If two states are each obligatory, then it is obligatory to have them both realised.
- Given principle (1) and our little story about papers we get:



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Inconsistent Obligations (II)

(2) $O(p \wedge \neg p)$

- Since standard Deontic Logics are extensions of standard logic, they include *ex contradictione qoudlibet*

(3) $\vdash (p \wedge \neg p) \supset q$

- As normal modal logics standard Deontic Logics contain their equivalent to the rule of necessitation

(4) $\vdash A \Rightarrow \vdash O(A)$

and the distribution equivalent to the (K)-Axiom

(5) $\vdash O(p \supset q) \supset (O(p) \supset O(q))$

- Given the normal character of Deontic Logic we get

(6) $\vdash O((p \wedge \neg p) \supset q)$

(7) $\vdash O(p \wedge \neg p) \supset O(q)$

and thus with (2) and *Modus Ponens*

(8) $\vdash O(q)$

for any sentence "q" whatsoever!

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Inconsistent Obligations (III)

- Thus having an inconsistent normative system – as it happens all the day – makes it obligatory to do whatever you like (burning the term paper, blowing up the university ...).
- This is unbearable. And that – fortunately – is not how we reason is these situations. In the example given there may be easy solutions, but in normative systems that have a long history the conflicts and the ways to resolve them are not so obvious (the existence of higher courts to solve such conflicts is witness to this).
- Given that we have not resolved the conflicts yet the underlying logic has to be a paraconsistent one.
- The question is, where paraconsistency has to intervene. Should the underlying logic of the non-deontic language (i.e. **PC**) be changed thus that we no longer have Explosion or similar rules? Or should the behaviour of sentences *within* the scope of deontic operators be changed, maybe by giving up the additivity of "O( )"?

p ∧¬p

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Standard Deontic Logic

- We ask ask ourselves if we want to drop any of the principles of standard Deontic Logic. These are:

  (DF) F(A) := O(¬A)

  (DP) P(A) := ¬O(¬A)

  (A1) ¬(O(p) ∧ O(¬p)) [Consistency of Obligation]

  (A2) O(p) ∧ O(q) ≡ O(p ∧ q) [Additivity of Obligation]

  (A3) O(T) [Inconsistency is forbidden]

  Rules: Uniform Substitution and *Modus Ponens*.
- Consistency assumption are built into standard Deontic Logic. By (A3) and (DF) we have that inconsistency is forbidden, and it is axiomatically demanded that we have no inconsistent obligations by (A1). Presumably both (A1) and (A3) have to be modified.
- In standard Deontic Logic we have a derived (K)-rule and its contrapositive:

  $\vdash (A \supset B) \Rightarrow \vdash (O(A) \supset O(B))$

  $\vdash F(B), \vdash (A \supset B) \Rightarrow \vdash F(A)$

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Standard PL Application to Obligations

- The former strategy of changing the underlying logic may be considered the standard application of a PL to questions of obligation.
- If the underlying logic of the normative language is changed to, say, **SKP**, then Explosion is no longer valid, and the argument given above does no longer go through.
- The principles we find in standard Deontic Logic don't have to be changed then.
- Syntactically this approach works by adding formation rules, axioms and rules for deontic operators to a paraconsistent logic.
  ["O(A)" with unneeded brackets dropped becomes "OA".]
- Semantically the typical truth condition of standard Deontic Logic for "O( )" can be taken over, even more so if the paraconsistent system has a modal semantic already.
- Note that this approach can deal with the occurence of O(A) and $\neg$O(A) as both true at the propositional level, since Explosion does not ensue by O(A) $\wedge$ $\neg$O(A).

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Standard Semantics of Obligations

- The paraconsistent treatment of deontic conflicts takes over the standard semantics of standard Deontic Logic. (This is the easier when the underlying paraconsistent logic has already a modal semantics.)
- One needs a (further) *deontic accessibility relation* between worlds thus that S(w,w') holds if w' is relative to w a *normatively optimal* world.
- "O(A)" is true at w if and only if in all worlds w' such that S(w,w') "A" is true in w' (... false iff in some world w' with S(w,w') "A" is false).
- If *permission* is not defined in the usual way [P(A) := ¬O(¬A)] one can give the basic truth condition as:
  "P(A) is true at w if and only if in some world w' such that S(w,w') "A" is true in w'. (... false iff in all worlds w' with S(w,w') "A" is false).
- Optimal worlds are intuitively those worlds where everything that should be the case is the case, and everything that is the case isn't bad.
- Note that in case of inconsistent obligation we have at least one *inconsistent deontic optimal world* w' with A and ¬A true at w' for some A.

Standard Semantics of Obligations (II)

- The deontic accessibility relation cannot be reflexive, since otherwise we had:

  $$OA \supset A$$

  which – unfortunately – is not the case.
- Iterations of deontic operators are also controversial. Thus it is questionable whether S should be transitive, since this leads to:

  $$OA \supset OOA$$

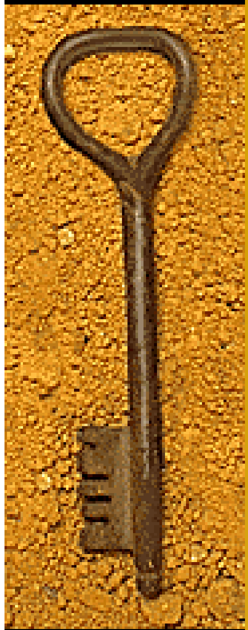
- The two basic conditions on S are that
  (i) S is serial: $(\forall w)(\exists w')S(w,w')$. Seriality means that no world is a deontic dead end. It makes

  $$OA \supset PA$$

  true. [Remember again the parallel to alethic accessibility conditions.]
  (ii) S is almost reflexive: $(\forall w,w')(S(w,w') \supset S(w',w'))$. Almost Reflexivity means that no new obligations are violated by doing what is obligatory. It makes true:

  $$O(OA \supset A)$$

  Something that should hold.



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Inconsistent Obligations in a Consistent World

- Casey McGinnis takes the later option of keeping the underlying standard logic: The basic idea is to have a logic that can deal with normative conflicts, but makes no further changes in logic, especially not at the non-modal level of ordinary sentences. So the world is considered to be consistent, therefore **PC** can be applied in its full strength. Obligations may be inconsistent, so the logic within the scope of deontic operators has to be a paraconsistent logic.
- This approach assumes:
  - the world is consistent
  - obligations may be inconsistent
  - additivity of obligations and other intuitive principles should hold
  - there may be gaps with respect to obligation (i.e. states such that O(A) is neither true or false, or at least that neither of O(A) and O(¬A) is true).

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Inconsistent Obligations in a Consistent World (II)

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- The idea of keeping the logic within the deontic operators apart from the underlying logic is modelled by keeping **PC** at the non-modal level and having **BN4** [see Chap.9] at the deontic level. The system is called **SPDL** (for "Semi-Paraconsistent Deontic Logic").
- **SDPL** like **SKP** singles out a "home world" w_0 as the starting point of accessibility. w_0 is required to be consistent and complete, i.e. atomic sentences receive either {0} or {1} as truth value. { } or {1,0} occur *only* at worlds different from w_0. Semantic consequence is truth preservation at w_0. "P" is not defined in terms of "O".
- The semantic rules have to ensure that inconsistency does not travel back from an accessible world *via* the rules of "O" into the home world: If a w_0-accessible world is inconsistent for A, then – according to the standard semantic rule for "O" given above – OA is true at w_0 *and* false at w_0, *and* O(¬A) is true at w_0. So there would be sentences with non-standard values at w_0.
- This problem can be solved by invoking a double standard in revising the falsity conditions of the standard semantics of "O" and "P".

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Inconsistent Obligations in a Consistent World (III)

- The revised rules are:
  (SO) $0 \in v(\text{"OA"},w)$ iff ($w = w_0$ and $1 \notin v(\text{"OA"},w)$) *or*
  ($w \neq w_0$ and $(\exists w')S(w,w')$ and $0 \in v(\text{"A"},w')$)
  (SP) $0 \in v(\text{"PA"},w)$ iff ($w = w_0$ and $1 \notin v(\text{"PA"},w)$) *or*
  ($w \neq w_0$ and $(\forall w')(S(w,w') \supset 0 \in v(\text{"A"},w'))$)
  [The parts for $1 \in v(\text{"OA"},w)$ and $1 \in v(\text{"PA"},w)$ remain the same.]
- Now one can prove that at w_0 *all* sentences are *bivalent*. (Look at the **BN4** truth tables for binary values and (S0) and (SP) above.)
- The Deduction Theorem holds for **SPDL**.
- Provable equivalents *cannot* be substituted for each other in **SPDL**, since in **PC** all contradictions are provably equivalent, but $O(p \wedge \neg p)$ need not say the same as $O(q \wedge \neg q)$, and the presence of deontic conjunction elimination would lead to deontic explosion.
- **SPDL** is not a normal modal logic, i.e. neither do "oughtification" for *any* tautology nor the (K)-rule hold: $\vdash_{\mathbf{SPDL}} \top$ and $\nvdash_{\mathbf{SPDL}} O\top$.

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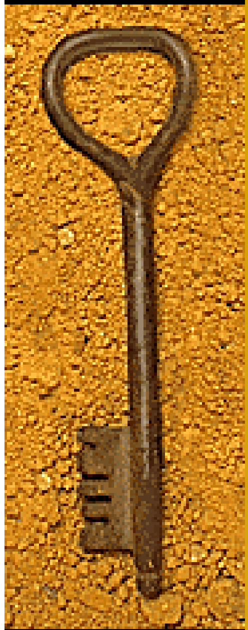
SPDL Theorems

- **SPDL** has as theorems:
  - (T1) $P(p \wedge q) \supset Pp \wedge Pq$
  - (T2) $Op \supset Pp$
  - (T3) $O(p \supset q) \supset (Op \supset Oq)$
  - (T4) $O(p \supset Pp)$
  - (T5) $OOp \supset Op$
  - (T6) $O(p \wedge q) \equiv Op \wedge Oq$
  - (T7) $P(p \vee q) \equiv Pp \vee Pq$
- **SPDL** invalidates:
  - (*1) $Op \supset OOp$
  - (*2) $Pp \supset OPp$
  - (*3) $Op \supset O(q \supset p)$
  - (*4) $O(p \vee \neg p)$
  - (*5) $F(p \wedge \neg p)$
  - (*6) $\neg O(p \wedge \neg p)$
  - (*7) $Pp \vee P\neg p$



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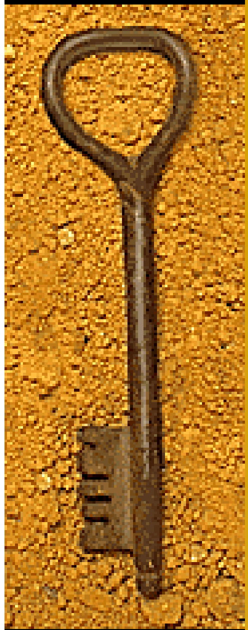
SPDL Explosion

- Note: Explosion holds in **SPDL** (since it has **PC** as underlying logic).
- Normative conflicts of the sort $OA \wedge O(\neg A)$ can be solved by **SPDL** and lead neither to deontic explosion nor to general explosion.
- If, however, we had $OA \wedge \neg OA$ this would be a case of an ordinary contradiction (at the home world) and would by the validity of Explosion allow to infer any B whatsoever!
- The occurence of this problem is softened by not having $PA \equiv \neg O \neg A$, such that the observation of a permission to $\neg A$ does not in conjunction with the norm to have A realized lead to everything. Thus the question is, whether we could ordinarily come into situations where we can ascertain that OA and can ascertain and $\neg OA$.
  It seems to me that we can: Imagine a situation where there are two sources of laws/rules, one demanding A the other being quiet on A.
- The absence of $PA \equiv \neg O \neg A$ also deviates from our ordinary notion of permission, as does the absence of $PA \vee P(\neg A)$.
- These may be reasons to have a paraconsistent logic also as the underlying logic in deontic reasoning.



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Questions

- (Q1) The case of inconsistent obligations, is that a case of weak paraconsistency or of strong paraconsistency, and what does the answer to this question depend upon?
- (Q2) What does Almost Reflexivity correspond to in alethic modal logic (respectively the logic of provability)?

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Exercises

- (Ex1) That some thing is forbidden means that its negation is obligatory. Consider the deontic necessitation rule

  $$\vdash A \Rightarrow \vdash O(A)$$

  Why does it say that is *forbidden* to realise *something inconsistent* (given the background of standard logic)?
- (Ex2) If in **SPDL** one did not revise the falsity condition of "O", but required $0 \in v(\text{"OA"},w) \Leftrightarrow 1 \notin v(\text{"OA"},w)$ – why does this make

  $$O(OA \supset A)$$

  invalid? [Draw an accessibility diagram.]
- (Ex3) Show for some of the **SPDL** theorems/non-theorems why they are valid/invalid.

Further Reading

- An introduction to Deontic Logic in general is: Aqvist, Lennart. *Introduction to Deontic Logic and the Theory of Normative Systems*. Neapels, 1987. The deontic "paradoxes" are also dealt with there, as well as Diadic Deontic Logic, which takes conditional obligation to be formalized by "O(A/B)" instead of "$B \supset OA$".
- A little overview on the basic systems and there semantics you find at:
  http://manuel.bremer.bei.t-online.de/SDL.pdf
  http://manuel.bremer.bei.t-online.de/DyDL.pdf
  (in German).
- Casey McGinnis' ideas are taken from his forthcoming PhD thesis *Paraconsistency and Deontic Logic*, (2004). McGinnis also gives a tableau style proof system for **BN4** and its deontic application.

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