

Assertion and Rejection

This essay deals with assertion and rejection. Its main question is whether rejection should be understood as a speech act *sui generis* in distinction to asserting the opposite.

§1 Some Motivation

If one asserts α [$\Vdash\alpha$]¹ one is committed to α . If one asserts $\neg\alpha$ one is committed to that. It might be that one is committed to both of them if the data (the evidence) are just thus. One may think of two sources of information on α , which come up with contradictory evidence. In this – supposedly unfortunate – case one supports α and one supports $\neg\alpha$. A data base which does not immediately trivialize on the presence of some inconsistent data would on request assert both α and $\neg\alpha$.

Now, in this situation if some person A asserts α with respect to an audience B, B cannot be sure that A does not also assert $\neg\alpha$ later (i.e. is inconsistently committed).

In some sense nothing can prevent that A follows up with $\neg\alpha$ a former assertion of α . A may be inadvertent or just inconsistent.

In some cases, however, a person A may not accept any old contradiction or take any old compromised evidence for real, but is committed to α and $\neg\alpha$ for some special α . This may happen in case of having to use some inconsistent theories (as was sometimes the case in the history of science, [cf. Meheus 2002]) or in the case of adhering to some doctrine of paraconsistency (like dialetheism: the claim that there are some α which are true and false at the same time [cf. Priest 1987]). Such a person *knows* that she is not just committed to α . If B knows that A (sometimes) is such a person, B really (i.e. apart from mistakes on A's side) cannot be sure what A's commitments are when A asserts α .

One way to deal with this could be an appeal to relevance. Person A should assert as much as she believes with respect to the content $\langle\alpha\rangle$, and thus should on being asked not just assert α

¹ The sign " \Vdash " is used here to symbolize assertion. One may use – as often in the literature – the sign of derivability " \vdash ". Inasmuch as assertion is an illocutionary force this may seem problematic, since an illocutionary force cannot occur within a sentence or more complex structure, but takes scope over the locutionary structures, and with a derivability sign one may have " $\gamma, \delta \vdash \alpha$ ". " \Vdash " is a pragmatic device whereas " \vdash " is syntactic and " \models " is semantic. These should be kept distinct.

but also $\neg\alpha$. Otherwise A would be misleading B. This, however, covers only the cases in which A at the time of the interchange *already* believes $\neg\alpha$. Person A may in other cases be ready to accept $\neg\alpha$ although having just asserted α , although not yet being committed to $\neg\alpha$. Can A in those cases in which A is *not* prepared to assert $\neg\alpha$ make this clear in advance?

(NAO) $\Vdash(\neg\alpha)$

i.e. not asserting the opposite of α , will not do, since this expresses only a present state (of mind). In general, talking about assertions and beliefs yields more beliefs which themselves have to be considered as assertable or not.

Therefore one may consider whether there is a stronger attitude than not asserting $\neg\alpha$. This attitude or *illocutionary point* may be: rejection $[\Vdash\alpha]$. To reject α shall be *defined* as (rationally) *excluding* asserting α .

(Once again it may be said that somebody may make mistakes and thus assert and reject some γ but this can always happen, just like the *inadvertent* contradiction $\Vdash\alpha$ and $\Vdash(\neg\alpha)$. No definition can prevent that something goes wrong with a human reasoner.)

Rejection allows, for a rational reasoner, to univocally commit themselves to α . They can have:

(CONS₁) One may uniquely commit oneself: $\Vdash\alpha$ and $\Vdash(\neg\alpha)$.

This combination *excludes* following up on $\Vdash\alpha$ with $\Vdash(\neg\alpha)$. It expresses *persistently consistent* attitudes towards α . Thus rejection $\Vdash\alpha$ is *stronger* than $\Vdash(\neg\alpha)$, which does not exclude following it up with $\Vdash\alpha$. We have three attitudes (illocutionary points) to distinguish:

- rejection $[\Vdash\alpha]$,
- asserting the opposite $[\Vdash(\neg\alpha)]$,
- withholding commitment $[\Vdash\alpha]$.

There is a slot in conceptual space for rejection once one accepts that there may be cases where $\Vdash\alpha$ does not enforce $\Vdash(\neg\alpha)$. An attitude with rejection's exclusive character has to have its place. We will explore here what conventions might govern a speech act of rejection and which logic it should obey.

§2 Usage

We need some general conventions of usage here. A sentence (like "Germany is north of Africa" or "I see a rabbit") is used in a declarative utterance to make a *statement* in some

situation, the full structure of which is something like ‘I believe/claim that ...’. The *situation* in which the sentence is uttered is a chunk of the (physical) universe. It often contains the speaker and the audience. The *context* of an utterance may include relevant but not or no longer present background conditions (like social position, prior history ...). The *content* of the statement is determined by the meaning of the sentence used as applied to the situation of usage and the context. By these rules indexical expressions (like “I”, “that” but also temporal order) are *anchored* to parts of the situation. The content of a statement can be given either by an eternal sentence (like “George Bush sees exactly one rabbit right in front of the left most window of the Oval Office on March, 26th, 2006”) or by a *state of affairs*. A state of affairs is an abstract set theoretical entity which represents what a statement claims by putting the *referents* of the expressions used into a tuple like <North-Of, Germany, Africa, 1>, where ‘1’ stands for a *polarity* expressing that the relation is supposed to hold (‘0’ being the opposite). A statement is *true* only if its content is an *obtaining* state of affairs, a *fact*. A fact is a structured part of the universe. State of affairs might be called *abstract situations* or *infons*.² Statements are thus taken as truth-bearers in virtue of their content. A statement is true only if its content is a fact. A statement consists in claiming that some sentence is true, respectively that some state of affairs obtains. One may judge or consider whether the illocutionary classification of an act (like ‘I assert that’) is appropriate, but evaluation in terms of truth concerns the content of a statement only. Statements properly translated share their content. A statement has a *situation of usage* and a *described situation*. Even with true statements these need not be the same. A true statement refers primarily to the fact described with it. The fact is its reference, its truth-value is its *evaluation*.

§3 The Classical Analysis of Rejection

Traditionally rejection and assertion of the opposite are not kept apart. This is due to a classical understanding of negation (“¬”). In (philosophical) reflections on standard logic and

² Whether there are abstract entities in a final metaphysical reduction may be doubted, but does not concern us here. For the ease of exposition the employed ontology at least involves sets and thus abstract facts. A comprehensive theory of truth will contain more than the mere necessary requirement of correspondence, but again that need not concern us here. The aim of this paragraph is not to offer theories or justified theses, but to lay down how some of the fundamental terms are used, so to avoid merely verbal controversies and misunderstandings that haunt the field. The terms are neither claimed to be used as in some of the classical texts where they play a major role or have been introduced. Neither are final statements about meaning, reference, speaker’s reference and so forth intended by following the usage outlined.

its way to formalize natural languages one does not introduce or justify an illocutionary mode *rejection* or *denial* besides the mode *assertion*, which is often taken to be represented by a sign like $\lceil \vdash \rceil$ or $\lceil \models \rceil$. A statement is taken as asserted to be true or – more often – just taken as given input to logical considerations. Assumptions in derivations are at least subjunctively supposed to be true. A negative mode like rejection or denial is declared as superfluous by understanding it along the lines of

(FREGE'S THESIS) Rejection is assertion of the opposite.

where the “opposite” of a statement α is its contradiction $\neg\alpha$. This assumption is seldom argued for. The *locus classicus* that one is referred to is Frege’s essay “Die Verneinung” (Frege 1919)³. Frege’s essay mainly deals with a couple of topics related to his conception of objective *thoughts*. Concerning the cited thesis Frege presents only *one* main argument (ibid p. 154)⁴, an argument appealing to simplicity (the more simple framework showing a more thorough analysis). Frege observes that a negation particle “not” (and related forms) has to be incorporated into a theory of thought *contents* in any case. Comparing then the two alternatives on assuming a further illocutionary force

a force of assertion	a force of assertion
a negation symbol	a negation symbol
a force of rejection	

the second is simpler, given the background of a standard understanding of negation (namely that exactly one of two contradictory statements is true). Given this background the situation cannot arise that we need a mode of rejection to do something which we could not do as well employing the mode of assertion (and using negation). Now, once this background of a standard understanding of negation and contradiction is put into question nothing is left of

³ The title of the paper already trades on a ambiguity in the German “Verneinung“. “Verneinung” can mean *negation* (i.e. the junctor or its natural language equivalents or the *negated sentence* as a whole), “Verneinung” can also mean *the act of denial/rejection* (i.e. *negating* something). Compare “ ‘Peter ist nicht zuhause’ ist die Verneinung von ‘Peter ist zuhause’.” for the first usage with “Peter verneinte, dass er gestern Abend zuhause war” for the second.

⁴ There may be a supplementary argument on p. 154 dealing with the scope of a supposed operator “it is wrong that” and its separation from the content of a thought, but the argument generalizes on just two example inferences presented and does not even consider possible ways to combine a logic of rejection with the presence of negation.

Frege's main argument. There may very well be situations in which rejection is not assertion of the opposite – whatever “opposite” may then be taken to mean.

Searle and Vanderveken in their *Foundations of Illocutionary Logic* are on the one hand securely within the camp of standard negation and *Frege's Thesis*. On the other hand their theory of illocutionary forces and illocutionary denegation provides room for a more complex analysis of negation and the relation of assertion and rejection (see below). Also some of those who proposed an independent speech act of assertion still subscribe to *Frege's Thesis* considered as an analytic equivalence claim (cf. Bendall 1979, Humberstone 2000).

§4 Withholding Commitment and Rejection

The distinction between $\|\vdash\alpha$ and $\|\alpha$ has a longer history than the distinction between $\|\alpha$ and $\|\neg\alpha$, it seems.

Reasons for withholding commitment on α have to be distinguished from reasons

- to believe/assert $\neg\alpha$

this requiring something like possessing positive counter-evidence, say about an alternative or some negative fact. (More on negative facts below.)

- to reject α

this involving commitment on content and compatibility conditions of α and $\neg\alpha$.

With respect to withholding there should be a distinction between withholding by being passive on α (passive withholding) and being outspoken on having no opinion on α (i.e. announcing non-commitment). Active withholding should include having reasons for this announcement. These reasons supposedly concern one's assessment of one's own epistemic situation. The content condition on these reasons might be something like

- having reasons that one does not hold beliefs concerning α

these reasons thus being *auto-epistemic* reasons.

§5 Rejection as a Speech Act *sui generis*?

If rejection is a speech act *sui generis* then there have to be constitutive principles/postulates for it. These correspond in function and content to the meaning postulates that govern assertion (as outlined in illocutionary logic by Searle and Vanderveken [1985]).

Still one may have doubts about rejection. Assertion carries a commitment to something being the case or being true. If rejection is contrasted to assertion one may suppose that rejection does not carry commitment. One who rejects does simply that. If rejection carried

commitment it might be cast as some form of assertion after all. So once again one had the reduction of rejection to some form of assertion. So the irreducibility of rejection seems to depend on rejection being non-committal.

On the other hand rejection has to have an illocutionary point: there is some purpose to be achieved by engaging in rejection. Commitment to this purpose should commit oneself to the purpose being realized, i.e. a fact to obtain. This need not be a commitment to a first level statement (about the world), but at least a commitment to a statement like

“One should reject that α ”

If rejection is rational, there has to be some reason for rejecting α . α being rejected has to depend systematically on some property of α . One may think of properties like α not being true etc. Properties of this kind correspond to assertions like “ α is true/not true”. In this case rejection depends on a preceding assertoric commitment at the first level. Then it seems one can achieve the purpose of rejection by a corresponding assertion like “ α is not true”, and one who rejects α is committed to that assertion anyway. Again the whole idea of having rejection as an independent illocutionary force collapses.

If the property of α that is the focus of rejection is not related to truth one may consider α being non-assertible. This second level property carries no immediate commitment to semantic properties, but again one has to ask *why* α is not assertible. This must have to do with the relation α has to the world, i.e. the semantic properties of α . So, even if the content of rejection should not be expressed as a semantic evaluation of some statement, it is systematically tied to some such evaluation which – if expressible – comes down to some assertion. (Inexpressibility of (semantic) evaluations being, of course, no option.)

How can these doubts about the independence of rejection be set aside?

Beginning with the last point, one may say that even if rejection is systematically tied to some evaluation of a statement (i.e. an assertion of such an evaluation) that does not exclude rejection from having an independent role in discourse. The meaning of rejection may underwrite that we can infer some evaluation of a statement from our rejection of it, but these consequences need not exhaust the role and concept of rejection.

Furthermore we may entertain the idea that there is nothing wrong with rejection entailing some semantic evaluation – what else should be the point of rejection? The role of rejection in a discourse may nevertheless be that there is a speech act that is stronger than asserting the opposite. As long as there is a specific role in illocutionary conceptual space to play for rejection there is nothing wrong with rejections having semantic content.

One may consider a theory on rejection as a substantial part in a theory of *illocutionary denegation*. The content of an illocutionary act may be negated or not. One can have two assertions one asserting the negation of the other. In distinction to this *propositional* or *sentential negation* illocutionary logic and speech act theory also deal with *illocutionary denegation*. Illocutionary negation is a relation between two types of speech acts/illocutionary forces. One of the two acts is the illocutionary negation of the other. The negation takes place in the force indication of the illocution, not within the (sentential) content. The two acts then can have the same content, but the one act is the illocutionary denegation of the other act. Candidates within the area of our investigation are assertion and withholding opinion (one pair) and assertion and rejection (another pair). Illocutionary denegation obeys certain laws of negation. Withholding one's opinion on α excludes asserting α , as does rejecting α , and vice versa. But whereas one need neither reject nor assert α or $\neg\alpha$, one by not asserting α and $\neg\alpha$ automatically withholds one's opinion on α .

The thesis of both rejection and assertion being present is a claim about natural language. So the justification of an basic illocutionary force of rejection cannot be that it is right now introduced into our account of language (or our scientific formal system used to model natural language). Of course, if such an introduction was possible, the resulting system might well serve its purpose. Why then, however, has this option been overlooked up to now?

One may say that the crucial phenomena have not been of major interest and our understanding of the corresponding pragmatic structure has evolved, although they have been in place all along (like we claim for recognized features and parameters in natural language syntax). One may say, as well, that the fruitfulness of employing such a distinction to account for undisputable properties of natural language (like the co-occurrence of both semantic closure and the felicity of discourse commitment) shows that something like this distinction between assertion and rejection has to have been in place all along.

A historical explanation of overlooking rejection may be that the study of speech acts is less than fifty years old, and the logic of speech acts – as are many semantic facts about natural language – is still under investigation.

§6 The Logic of Rejection

If it is possible to have $\Vdash\alpha$ and $\Vdash(\neg\alpha)$, and $\Vdash\alpha$ and $\Vdash\alpha$ are stronger incompatible with each other than the first pair, then

$$(*) \quad \Vdash(\neg\alpha) \quad \Rightarrow \quad \not\Vdash\alpha$$

cannot hold. Just because we assert $\neg\alpha$ one cannot infer that we not also assert α .

What about

$$(**) \quad \not\vdash\alpha \quad \Rightarrow \quad \not\vdash(\neg\alpha)$$

If rejection is the stronger attitude this should hold, one might think.

The use of rejection was introduced as allowing an audience to hold that

$$(RW) \quad \not\vdash(\neg\alpha) \quad \Rightarrow \quad \not\vdash\vdash(\neg\alpha).$$

This is weaker than (**), however. Whether (**) should hold depends on the commitments bound up with rejection and assertion. If assertion means to be committed to give further justification – as it does according to illocutionary logic – or to claim the obtaining of some fact corresponding to the statement, then assertion may have different commitments from rejection:

- (i) With $\not\vdash\alpha$ one commits oneself to the claim that the justification and/or the fact corresponding to α are missing.
- (ii) With $\not\vdash(\neg\alpha)$ one commits oneself to the claim that the justification and/or the fact corresponding to $\neg\alpha$ are given.

These commitments need not be identical. Thus being committed to $\not\vdash\alpha$ need not commit us to $\not\vdash(\neg\alpha)$. Thus (**) does not hold.

Ross Brady has proposed a logic of rejection (cf. Brady 2004). In this logic theorems are asserted and non-theorems (although these may be simply true sentences) are rejected. The logic has standard axioms and rules for the propositional theorems, and adds for rejection:

Rejection Axiom: $\not\vdash p$

Rejection Rules: $\not\vdash\alpha \supset \gamma, \not\vdash\gamma \Rightarrow \not\vdash\alpha$
 $\not\vdash\alpha^{y/p} \Rightarrow \not\vdash\alpha$

The rejection axiom says that no simple sentence is a theorem, and thus should be rejected.⁵ The first rejection rule says that if the consequence of some sentence should be rejected, then the sentence itself should be rejected, the second says that if some sentence has to be rejected under uniform substitution it should be simply rejected.

Interestingly enough the standard theorem schema $\alpha \wedge \neg\alpha \supset p$ and the second rejection rule with the rejection axiom entail: $\not\vdash\alpha \wedge \neg\alpha$ (i.e. a form of the *Law of Non-Contradiction*). Another simple derivable rule says

⁵ “p” is a single sentence of the system, not a schema; otherwise everything would be rejected.

$$(RNC) \quad \Vdash \alpha \quad \Rightarrow \quad \not\vdash (\neg\alpha)$$

thus enforcing consistency of theoremhood. A system that does not allow for the overlap of the set of theorems and the set of non-theorems is, of course, too strong for the situations mentioned in §1. In the light of the illocutionary acts set apart also in §4 the rejection axiom seems appropriate rather for withholding assent [\Vdash] than for rejection. The rejection rules outlined can be accepted even by a dialetheist: if some theory has consequences to be rejected, so should be that theory.

§7 The Metaphysics of Rejection and Asserting the Opposite

Rejection concerns the conviction that some fact is *absent*. Assertion of the opposite in distinction claims some fact to be present, namely *a negative fact*. The distinction between $\Vdash \alpha$ and $\Vdash (\neg\alpha)$ depends, at least in some related metaphysical conceptions, on making a distinction between the absence of the fact $\langle\langle\alpha,1\rangle\rangle$ and the fact $\langle\langle\alpha,0\rangle\rangle$, respectively $\langle\langle\neg\alpha,1\rangle\rangle$, this later distinction depending on whether one allows for truth-value gaps or not. If for some α $\neg\alpha$ is more than the absence of $\langle\langle\alpha,1\rangle\rangle$ then it may be the case that one is neither committed to $\langle\langle\alpha,1\rangle\rangle$ nor to $\langle\langle\neg\alpha,1\rangle\rangle$.⁶ One may reject both corresponding sentences.

That the fact $\langle\langle\alpha,1\rangle\rangle$ does not exist/does not obtain need not imply that another fact $\langle\langle\neg\alpha,1\rangle\rangle$ of that level entities does exist. Negative facts might be introduced as higher order facts: If on some level n $\langle\langle\alpha,1\rangle\rangle$ is missing one may note on level $n+1$ that this fact is missing. Such an higher order fact would roughly look like $\langle\langle\neg, \langle\langle\exists, \langle\langle\alpha,1\rangle\rangle, 1\rangle, 1\rangle\rangle$. In many cases (typically in case of physical facts) the negative fact is just such a higher order fact, and nothing else. These negative facts are supervenient on the other (i.e. positive) facts, and thus are tied to their positive states of affairs. In these cases we have

$$(WAO) \quad \Vdash \alpha \quad \Rightarrow \quad \Vdash (\neg\alpha)$$

In some cases, however, the negative fact may be independent from the positive facts. In these cases the negative fact need not be a higher order fact. In these cases $\langle\langle\alpha,1\rangle\rangle$ and $\langle\langle\neg\alpha,1\rangle\rangle$ may exist at the same level. In acknowledging both these facts one can come to believe α and $\neg\alpha$, thus $\Vdash \alpha$ and $\Vdash (\neg\alpha)$. Since negative and positive facts of such an inconsistent level are co-assertible we can have informative inconsistencies.

⁶ But even if commitment allows for gaps that does not imply that truth also allows for gaps.

We do not have (WAO) because for some α the fact corresponding to $\neg\alpha$ may be *more* than the absence of $\langle\langle\alpha,1\rangle\rangle$. Thus by withholding assent to $\langle\langle\alpha,1\rangle\rangle$ one is not automatically committed to $\langle\langle\text{NOT},\alpha,1\rangle\rangle$.

We may consider some variants of negative facts to make this clear:

- (i) Negative facts as absence of positive facts are like Sartre’s famous example: Pierre is not in the café. The totality of positive facts in the café does not include Pierre’s presence.
- (ii) Negative facts may be equivalent to positive facts that exclude other positive facts, like: The handkerchief is not dry \equiv The handkerchief is wet.
- (iii) Negative facts may be opposed to a positive fact although not allowing to *infer* to its absence. If we allow for an infinite conjunctive fact corresponding to $\neg F(0)\wedge\neg F(1)\wedge\neg F(2)\dots$ this fact is opposed to the positive fact corresponding to $(\exists x)F(x)$, although the negation of $(\exists x)F(x)$ cannot be inferred by us, since the ω -rule is not feasible for us.
- (iv) Lastly, negative facts may be co-existing with their positive facts. The proof of the Liar antinomy’s truth $[\Vdash T\lambda]$ could be, for example, the fact corresponding to it. The fact behind $\neg T\lambda$ could be the proof of it *not* being true in naïve semantics, which, of course, is not the absence of the first proof. Another example might be provided by the properties of inconsistent objects. Concerning the round square we have:

$\langle\langle\text{ROUND},\text{ROUNDSQUARE},1\rangle\rangle,\langle\langle\text{NOT},\langle\langle\text{ROUND},\text{ROUNDSQUARE},1\rangle\rangle,1\rangle\rangle,$
 $\langle\langle\text{SQUARE},\text{ROUNDSQUARE},1\rangle\rangle$ – and so on.

In the case of interesting inconsistent information we have negative facts of kind (4). The consistency enforcing principle (CONS₁) uses rejection to make clear that the case α under discussion is not of this kind.

In the case of Liar-like antinomies the facts consist in the negation of a sentence being as provable as the sentence itself. There is no further fact “behind” this. Since the proof is an existent *something* one may even speak of a positive fact here, like the intuitionist bases the claim for $\neg\alpha$ not on the absence of reasons for α , but on the (positive) proof of \perp from the premise α (cf. Priest 1987: 87).⁷

⁷ The case is different with the set-theoretical antinomies, since here we seem to have the negative fact of the Russell set not belonging to itself besides the (positive) fact of the Russell set belonging to itself. These negative facts – one may argue – have their residue in

§8 Unique Commitment

If rejection is a speech act set apart from asserting the opposite, one can consistently commit oneself to α being only true by

$$(1) \quad \Vdash \text{True}(\alpha) \text{ and } \not\vdash (\text{False}(\alpha))$$

or shorter:

$$(1') \quad \Vdash T(\alpha) \text{ and } \not\vdash F(\alpha)$$

For (1) to be true, α has to be true and rejecting the falsity of α can be successful only if α is not false; thus α is true only.

Thus in case of consistent statements one can uniquely commit oneself to a single truth value of such a statement.

What about the antinomies, like the Liar? The Liar is considered by the dialetheist to be both true and false. So the dialetheist should claim

$$(2) \quad \Vdash T(\lambda) \text{ and } \Vdash F(\lambda)$$

Since falsity does not – for the dialetheist – exclude truth (2) itself is not of the form (or implying a sentence of the form): $\gamma \wedge \neg\gamma$.

But a lot now depends on the semantics of the expressions “true()” and “false()”. Paraconsistent logics can level the distinction between object and meta-language. A semantically closed language not only is able to talk about its own expressions, but does contain at the same time its semantic expressions. These semantic expressions need not be taken as predicates (like a truth predicate applying to the quotation of a sentence), but can be taken as operators instead. One arrives at a paraconsistent language/logic which allows truth value talk without previously quoting the sentences which are evaluated.

Are these bivalent operators following the table

α	$\neg\alpha$	$T\alpha$	$F\alpha$
0	1	0	1
1	0	1	0
0,1	0,1	1	1

the realm of abstract objects, however; much may be going on there. (If one takes sets not as abstract entities dialetheism in set theory may be a problem for realists.)

or do they mirror the paraconsistent evaluations, like in

α	$\neg\alpha$	$T\alpha$	$F\alpha$
0	1	0	1
1	0	1	0
0,1	0,1	0,1	0,1

In the first case (2) is a simply true statement, equivalent to

$$(3) \quad \Vdash T(\lambda) \wedge F(\lambda)$$

In the second case, however, a conjunction like (3) is a dialetheia itself. The main thesis of dialetheism (claiming a contradiction to be true) would turn out to be dialethic itself. This seems to be devastating for a philosophical theory. Even if there are occasions on which it is rational and relevant to utter a contradiction one may assume that theories should not be contradictory in this sense – at least one not intend and know that they are contradictory in this sense. Therefore the operators for truth values should be taken as bivalent.

§9 Truth Operators as an Alternative to Rejection?

We just argued that semantic evaluations should better be bivalent. This might lead to an approach that looks for something else to the role of rejection, as outlined above. This something would have to carry some commitment to a semantic evaluation, which nonetheless is incompatible with asserting α , *even* if one allowed for some inconsistency.

One option in this direction is the usage of truth operators in a semantically closed language, employing some paraconsistent logic. Such truth operators are used in assertions only, but some of these assertions may play the role that rejection as outlined above is proposed to play.

One may use bivalent truth operators working in the fashion of the following table:

α	$\neg\alpha$	$T\alpha$	$F\alpha$	$\Delta\alpha$	$\nabla\alpha$	$\circ\alpha$	$\bullet\alpha$
0	1	0	1	0	1	1	0
1	0	1	0	1	0	1	0
0,1	0,1	1	1	0	0	0	1

“ $\Delta\alpha$ ” says that α is true only, “ $\nabla\alpha$ ” that α is false only, “ $\circ\alpha$ ” says that α is consistent (i.e. has only one truth value), “ $\bullet\alpha$ ” says that α is contradictory. We can then say – and these being

just true – that the Liar is true, false, not simply true, not consistent, and so on. “T” and “F” are now understood as operators applying to sentences not to quoted sentences. Some of these operators are incompatible with each other (their columns in the table being the opposite of each other): $T\alpha$ and $\nabla\alpha$, $F\alpha$ and $\Delta\alpha$, but not $T\alpha$ and $F\alpha$.

The trivialist proposes $(\forall\alpha)(T\alpha \wedge T\neg\alpha)$ or $(\forall\alpha)(T\alpha \wedge F\alpha)$. The dialetheist claims $(\exists\alpha)(T\alpha \wedge F\alpha)$, but also $(\exists\alpha)(T\alpha \wedge \neg T\neg\alpha)$, and $(\exists\alpha)\nabla\alpha$. And given some corresponding formal system some formulas can be exhibited having these properties (e.g., defining a bottom particle \perp with $\nabla\perp$ being valid). \top can be defined as the top particle with $T(\alpha \vee \neg\alpha)$, being true only. The bottom particle \perp can be defined as $\nabla(\alpha \vee \neg\alpha)$, being false only. Note that – in contrast to even the intuitionist negation rules – $\perp \equiv (\alpha \wedge \neg\alpha)$ need not hold if α is a dialetheia, since then $T(\alpha \wedge \neg\alpha)$, and ∇ is incompatible with T .

To have and use the Convention (T) at the same time as these operators (be it for the operator “T” or “ Δ ”) one needs some revisions in the logic of the conditional, like giving up on the unrestricted validity of Contraposition. $T\alpha \wedge \nabla\alpha$ is a well-formed formula, but false only. The language of this version of dialetheism thus contains formula that can be evaluated only as being simply false. These formulas, of course, cannot be derived.⁸

Thus with respect to ordinary sentences (the truth of) $\neg\alpha$ may be the absence of (the truth of) α , but if α entails $\neg\alpha$ and vice versa, and both are of interest, given that there is independent ground for $\neg\alpha$, accepting $\neg\alpha$ does not exclude accepting α . In contexts we know to be consistent we may reason to $\neg\alpha$ without independent grounds on the basis of $\neg T\alpha$ and *tertium non datur* (or some version of this disjunctive syllogism like reasoning). Since in case of antinomies accepting $\neg\alpha$ does not exclude accepting α , accepting $\neg\alpha$ should not be the same as rejecting α .

⁸ We do not need the details of all these restrictions here. The reader has only to know the general idea of paraconsistent logics and the idea of “adaptive logics” (Batens 1989, 2000) to restrict some rules to consistent sentences (respectively to retract some supposed consequences if the rules to derive them employed, against the restrictions, some inconsistent sentences). A paraconsistent logic like Priest’s **LP** (Priest 1979) can be developed into an adaptive logic with a restricted form of *Modus Ponens* and Contraposition (Priest 1991). Within paraconsistent logics “logics of formal inconsistency” (Marcos 2005) employ consistency operators in the object language. Truth operators can then be added. Blending these approaches one can have an adaptive paraconsistent logic which combines the extensional and intuitive truth conditions of **LP** with the use of truth and consistency operators (Bremer 2005). We suppose here that the dialetheist uses some such logic. On the whole idea of asserting and believing contradictions making use of these truth operators cf. (Bremer 2007); “rejection” in the sense employed here is called “denial” there.

Rejecting α [$\not\vdash\alpha$] can then be understood by a dialetheist as affirming $\nabla\alpha$ [i.e. $\Vdash\nabla\alpha$]. Rejecting α would thus be incompatible with affirming α (i.e. affirming $T\alpha$).

Whereas there are situations in which a dialetheist accepts both α and $\neg\alpha$, there are no situations in which a dialetheist accepts and rejects α at the same time. Dialetheism does not accept just any contradiction. This is one reason – prejudices and puns to the side – why rational argument with a dialetheist is possible. As the foregoing distinction shows there is, furthermore, one kind of contradiction that (even) a dialetheist cannot support:

$$(\text{CONS}_2) \quad \neg(\Vdash\nabla\alpha \wedge \Vdash T\alpha)$$

since $T\alpha$ and $\nabla\alpha$ are semantically incompatible.

Another simple point is that no-one (including the dialetheist) can have pragmatic contradictions: Speech acts being bodily movements that either occur or do not, there is no pragmatic parallel to having it both ways, i.e.

$$(\text{CONS}_3) \quad \neg(\Vdash\alpha \wedge \Vdash\neg\alpha)$$

This instance of the accepted tautology $\neg(\alpha \wedge \neg\alpha)$ expresses not only a semantic exclusion the dialetheist accepts (and sometimes nevertheless supersedes), but the absence of the mysterious feat of asserting something and not doing it at the same time. There is no pragmatic dialetheism (without a verbal manoeuvre of redefining “not asserting” on the lines of “asserting $\neg\alpha$ ”).

Rejection thus understood comes close to the standard understanding we saw in Frege. The main difference is that more than one negation is present in the language. Whether this stronger negation “ $\nabla\alpha$ ” leads to new paradoxes is an open question. So far it seems not, because of restrictions of the logical rules when dealing with inconsistent sentences.

§10 Rejection Again

Even if there are no (decisive) arguments against the just outlined truth operator approach one may think that the idea of rejection as a speech act is also viable, or may even be preferred as more natural.

One of the conditions of rejecting α is:

- to have a reason excluding asserting α

This is the commitment condition of rejection that corresponds to the conditions of having reasons for α when asserting α . Expressed in this way rejection would be a meta-attitude, since it refers back to another attitude (namely asserting α). Expressed as a lower level attitude it would be like

- to have a reason excluding the truth of α

Such a reason need not be a reason to assume the truth of $\neg\alpha$. And it need not be weaker. The paradigmatic reason for excluding the truth of α may be: having asserted $\neg\alpha$ (i.e. believing in the truth of $\neg\alpha$) *and* believing α to be consistent.

This shows that rejection occurs in contexts supposed to be consistent. α may *be* consistent or *believed* to be consistent. It might be a *shared belief* in a situation of utterance that α is consistent (i.e. might be part of the context).

A principle corresponding to (CONS₃) would then be

$$(CONS_4) \quad \neg(\|\neg\alpha \wedge \|\alpha)$$

Since rejection is contrary to assertion one cannot (rationally) assert and reject the same sentence (content).

At the end of an exploration of rejection as a speech act *sui generis* one may claim dialetheism to be rationally maintainable in a discourse by making use of the distinction between rejection and non-assertion. Unique commitment is possible for a dialetheist. By outlining

- the parallels to assertion
- the basic principles of a logic of rejection
- the epistemological situation of one who rationally rejects some α
- some ontological distinctions concerning negative facts

rejection might be considered a notion clear enough to carry that burden.

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