# Concepts and Objects – What Else to Say?

Is Frege's Ontological Dualism Inexpressible?

§1. Frege considered his distinction between functions/concepts and objects as one of his main achievements. He employs it within his theory of sentential unity. General terms expressing concepts are unsaturated linguistic item which in combination with singular terms yield again saturated linguistic items (sentences). Filling the argument places of general terms distinguishes the built up of sentential unity from a mere list. This combination on the linguistic level has a corresponding combination on the level of reference. Functions (and concepts as functions from objects to truth values[1]) are unsaturated entities and their combination with objects yields saturated entities again. Frege thus asserts both a linguistic categorical dualism as well as an ontological categorical dualism. The entities in the two categories are, tautologically, categorically distinct – but can we say so? Frege himself (in)famously claimed that

(1) The concept *horse* is a concept.

is not true as the expression “the concept horse” is a singular term and thus designates an object! How can we speak then about concepts at all? Similarly

(2) No concept is an object.

which looks like the expression of Frege's ontological categorical distinction cannot be true, or not even be properly built, as the supposed logical form of it

(3) $\forall x\,(\text{Concept}(x) \supset \neg\text{Object}(x))$

treats concepts and objects as being of the same underlying (neutral) ontological category. Again concepts are assimilated to objects. If this is ontological and syntactic nonsense, how can Frege's ontology be expressed at all? Can the difference between concepts and objects only be *shown* in a

1 In the following I speak mainly of concepts. The same story could be told about functions.

properly regimented language like Second-Order Logic (SOL)? If that was the case, it would have wide philosophical repercussion concerning the possibility of meta-logic and meta-theory in general, as witnessed by Wittgenstein's *Tractatus*. Frege, at times at least, seems to settle with inexpressibility.[2]

§2 In a corresponding meta-language one may speak about the expressions and their syntax. Truth conditions and semantic modeling show the categorical differences of the types. Nevertheless two problems stay with us:

(I)

Distinct statements are used to characterize the distinct types, which reveals again the type distinction (e.g. “If $\alpha$ is a general term ….”). Only if we have talked so far only about syntactic composition, everything sounds proper. This poses no problem as expressions *are* objects, and thus can be covered by a common category. We may even try to express truth conditions using schemata. This raises the issue what we understand when we understand schemata – supposedly a generalization about the involved types of entities.

(II)

Attempting ontological type neutral meta-language talk we presuppose an understood type system of the meta-language. We may end up in a hierarchy of meta-language explanations of type distinctions.

In any case we have not expressed (2), Saying of two types A and B that they are distinct

(4) $\neg\exists x(x\in A\wedge x\in B)$

again uses a domain of entities of neutral type. If x has a concept as value and concepts are

2 “Wenn es irgendetwas geben kann, was nicht Gegenstand ist, so kann man diese Thatsache nicht ohne Widerspruch aussagen.” Letter to Russell, June 24th 1902 (cf. Gabriel, Gottfried/Kambartel, Friedrich/Thiel, Christian (Eds.) *Gottlob Freges Briefwechsel*. Hamburg, 1980. Similar passages can be found in other letters to Russell. Frege explicitly resorts to semantic ascent: “Statt des ungenauen Ausdrucks '$\xi$ ist eine Funktion' kann man sagen '( )*3+4' ist ein Funktionsname.” (Letter to Russell, June, 29th, 1902).
Also cf. Ricketts, Thomas. “Concepts, objects and the Context Principle”, in: *The Cambridge Companion to Frege*, ed. by Michael Potter and Tom Ricketts, Cambridge (CUP), 2010, pp. 149-219.

essentially unsaturated, then $x \in A$ cannot be a thought/proposition *at all*.

Accordingly: If “Object( )” in (3) was higher order (thus being able to take concepts as arguments) we might get a proper thought/proposition, but one which does no longer say what it was supposed to say, as its content becomes more or the less equivalent to

(5) $\forall x\ (\text{Concept}(x) \supset \text{Concept}(x))$

Objects drop out of the statement. (2) is not expressed.

§3 A completely universal quantification had to cover both concepts and objects. A general concept of 'entity' might serve this purpose. (3) and (4) would be all right then, quantifying over entities in general. Our problem reoccurs with the syntactic typing of the variables used then. We needed a neutral syntactic category as well. Concepts and objects would be values of variables of this type. Then expressions denoting values of variables of this type could be either general terms or singular terms. If we had the neutral (syntactic) category we could built sentences with combinations of concepts expressions and object expressions which are not intuitively well-built (say, as they switch the position of singular and general term). Given Frege's theory of the semantics behind sentential unity some combinations of 'entities' just don't suffice.

Should one even allow such combinations and postulate that they are all to be evaluated as false? As false they could still occur as consequence or premisses of inferences, which makes no sense. Even though it is true that non-well-formed expressions aren't true. This might be linked to a general principle:

(P) Syntactically non-well-formed expressions cannot be true.

(P) is obviously correct, but does not solve the problem of expressibility either.

Pushing this observation from the meta-language into the language – a paraconsistent language as we now are able to use a truth predicate or truth operators within a language – we get (given disquotation and the opposition of truth and falsity) from “False(“Peter(( )is a horse)”)” to the syntactic nonsense “¬Peter(( )is a horse)” again.

§4 We seem to come closer to expressibility by semantic ascent. We might say, for instance:

(6) That which “( ) is red” denotes is a concept, and we cannot say of a concept that it is an object.

We still cannot express (2) in its generality. In its intended syntactic reading the second conjunct of (6) asserts the impossibility of a well-formed sentence of some structure. We can *express* our inability to express an ontological truth, and we cannot express the ontological truth itself. We cannot even express why we cannot express this ontological truths, it seems.

Our next attempt in ascent might be:

(7) $\forall x$ (ConceptWord(x) $\wedge$Welldefined(x) $\supset$ $\exists$F (Denotes(x,F))

Now, we need to introduce *two concepts of denoting* as the one takes a second-order argument and the other a first-order argument! This may seem unfortunate as denoting should be the same relation for all expressions, but it may still be so, our theory of denoting may explain as much. Only our relations of denoting have for categorical reasons to be as many as there are categories.

The truth condition of 'saying of ( )' may then state that one may *say* a general term $\alpha$ *of* some entity x which is denoted by $\beta$ iff $\alpha(\beta)$ is syntactically correct, talking about linguistic entities only. Denoting should get us to the referent of the linguistic item, thus “what '( ) is red' denotes” should be an expression getting us at the concept expressed (the *referent* of “( ) is red”). We may, following Frege in some of his remarks[3], say even:

(8) The book is what '( ) is red' denotes.

This seems a bit cheating, however, as “what '( ) is red' denotes” still is a singular term, and, therefore, should, by Frege's own original argument, be denied to denote a concept. All the proper work done in (8) is done by “( ) is ( )*”. “My pencil what '( )is red' denotes” is no sentence.[4]

In this mood of linguistic ascent we may further proceed to

(9) $\forall$F (“Object( )” cannot be said of F)

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3 Cf. Frege, Gottlob. *Schriften zur Logik und Sprachphilosophie*. Aus dem Nachlass. Hg. v. Gottfried Gabriel. Hamburg, 1971, S.31. A never published manuscript.

4 Dummett (*Frege*. Philosophy of Language. Cambridge (Harvard University Press), 2nd Ed., 1981, pp. 213-14) follows Frege here and papers over the role of “is”, which cannot be left out of the sentence.

and correspondingly for objects and “Concept( )”. No concepts can be said to be an object as that combination cannot be syntactically correct: “Object( )” should take *first-order* arguments.

We still cannot say *why* some combinations of signs are semantically incorrect. We can stipulate that they are syntactically incorrect, but such stipulations ultimately depend on a semantic picture (like Frege's picture of concepts as unsaturated).

Frege's demand for determinacy of concepts

(DET) $\forall F\ \forall x\ (F(x) \vee \neg F(x))$

is a formula satisfied by concepts only, and thus might be seen as defining 'concept' and the corresponding category, the open formula (without the universal quantifier on concepts) being true only of concepts, which, however, is enforced by the syntax of the argument in the first place.

§5 Given a theory of extensions or sets we may define in our meta-language

(10) $B = \{x \mid x \text{ is denoted by a general term}\}$

(11) $G = \{y \mid y \text{ is denoted by a singular term}\}$

and as extensions/sets are first-order entities we can now say

(12) $B \cap G = \varnothing$

A set theory like ZFC allows us to say as much, but ZFC itself introduces a categorical distinction concerning the status of the relation [$\in$] which has no extension in ZFC. ZFC itself works with an ontological categorical distinction between its sets and non-set collections (like the universe *V* itself). The debate would change only in details if we considered this distinction. ZFC gets rid of concepts in favour of sets, but contains its own inexpressible ontological duality, even if it is only presupposed. Additionally we may nonetheless need a theory of concepts as part of our theory of sentential unity.

§6 The paradox of the concept *horse* points to a fundamental dilemma: Either we do not distinguish semantic types (i.e. types in the reference of expressions) and cannot explain sentential unity, or *we do so* being able to explain sentential unity, but have traded in the problems of expressibility. The difficulties of hidden type distinctions in ZFC point to a similar dilemma.

§7 Frege allows for mixed-level general terms (the most well-known being "( )is the value range of ( )*"). And even if Frege had not done so, there is no principled objection against mixed-level general terms. If mixed-level general terms are allowed we may introduce the predicate "categorically distinct" and say:

(13) $\forall$x $\forall$F (x is categorically distinct from F)

To introduce the predicate we have to stipulate the first argument to be first-order and the second argument to be second-order. This can be said as we talk *in this stipulation* about linguistic items. What we have not done – still – is to explain what "categorically distinct" means. Once we move towards an explanation we enter ontology again and face the problem of (2).

Using a device like (13) we can justify and establish our symbolism (say in SOL). A statement like (13) repeats what is shown by the use of different types of variables. Note, however, that (13) does not talk about linguistic items, but about concepts and objects themselves! We may achieve more than what simple semantic ascent can give us. Ascent elucidates our usage of different variables and expressions, whereas (13) reaches the ontological level. Therefore we also not just deal with a 'saying/showing' type of elucidation here. And the recurs to semantic ascent provides no necessity here to drop ontology in favour of philosophy of language. What is left open is an ontological thesis corresponding our linguistic thesis. (2) cannot be said – but should we bother? We can say that concepts and objects are categorically distinct and this concept of categorical distinction is embedded in our general theory of language and ontology. Some concepts have to be basic, so why not 'categorically distinct'?

§8 So, how far does inexpressibility go? To express (2) may be an expectation stemming from our ordinary distinction of categories, which should not be taken over lightly to fundamental matters. So – maybe – (13) is all we get, and all we need. On second sight a problem of expressibility reoccurs. (13) allows us do distinguish objects and functions/concepts *of some* *definite order*: the second argument of the expression “( ) is categorical distinct from ( )*” has to be of some specific order (say allowing to apply the expression to functions/concepts of objects), and thus does not allow its application to functions/concepts of the next order! So we may say, for instance, that concepts of objects are categorically distinct from these objects, but then what about second-order concepts? We seem to need another expression “( )is categorically distinct from ( )*” with the arguments being second- and first-order functions/concepts – and then another one...

We land in a hierarchy of such concepts and statements of categorical distinction, corresponding to Frege's hierarchy of functions/concepts. Even with a basic predicate and concept of categorical distinction we cannot express *in general* (i.e. across the whole hierarchy) that concepts are categorically distinct from objects (and from each other according to their order). Strict distinctions of type and order present the dilemma – besetting also Russell's type theory or other hierarchies like Tarski's semantic hierarchy – that either some features of the structure of the hierarchy are inexpressible or in our attempt to express them in conveying the hierarchy we land ourselves in performative inconsistency (doing what our theory says cannot be done).

For Frege the regress of ever more levels of “( ) is categorically distinct from ( )*” might be seen as virtuous instead of vicious, as we may resort to such statements when needed, and above the third level there are no crucial applications of such statements, given that Second Order Logic is all we need. A shortcoming of such a proceeding is that we either introduce the expressions of our formal language (namely the statements of category distinction) piecemeal or by a generic statement/schema about expression of the general form “( ) is categorically distinct from ( )*”, which cannot be part of our theory itself again. We need ascent to a richer meta-language then, which gets us into conflict at least with the conception of logic being completely universal. As Frege sometimes employs meta-logical arguments, he might have accepted this form of ascent, and it certainly is common practice today.

§9 Assuming Frege's approach struggles with problems of expressibility, can we keep Frege's account of sentential unity and allow concepts to be referred to? Once we allow for ascent to a hierarchy of concepts and a meta-language, there seems to be a solution. We have to combine Frege's thesis on sentential unity with the claim that concepts can be subjects of sentences and can be referred to by (special) singular terms. For a theory of sentential unity we need the claim that in the constituent structure of a sentence we need two categorically distinct components, distinct in their syntax and semantics. Frege's distinction between concepts and objects provides just that. Frege's error lies in the move from the proper claim that some unsaturated expressions need to refer to unsaturated entities, and that some saturated expressions need to refer to saturating entities, to the improper generalization that no saturated terms can have unsaturated entities as their referents. The syntactic/semantic-isomorphy on unsaturatedness is unwarranted.

The account then may be this:

(SU) Theory of Sentential Unity

i. The unity of sentences involving first-order general terms (referring to first-order concepts) and singular terms referring to objects stems from the general terms (and the concepts they refer to) being unsaturated, them being saturated at their argument positions by singular terms referring to objects, which are saturated (as are their referents).
ii. Singular terms of the form "the concept ___" refer to concepts.
iii. Singular terms of the form "the concept ___" with the concept filling the slot being of order *n* saturate general terms referring to concepts of the next order, *n+1*.

This account differs from Frege's in introducing a hierarchy of singular terms corresponding to the hierarchy of concepts, but keeps the essential account of sentential unity. Frege himself in employing Second Order Logic allows for concepts being the arguments of higher-order concepts so that their own unsaturatedness does not make a sentence/thought about them unsaturated, once the higher order concept has been saturated by them. Clauses (ii) and (iii) in (SU) make generic claims about concepts *in general* (i.e. across the whole hierarchy of concepts), and thus have to be made in a meta-language, which ultimately faces the same problems itself, giving rise to another meta-language – and so forth, as it is with these hierarchies of meta-languages. This undermines the idea of logic as truly universal, but cannot be held against a Fregean account by those who employ similar hierarchies in their own approaches.

§10 If you are dissatisfied with the virtuous or vicious regress to meta-languages, the ultimate solution could be to drop the distinctions between orders of concepts and use one level of concepts only, i.e. forsake any hierarchy in the objects or in the concepts. This would allow for a simplified theory of sentential unity.

(SSU) Simplified Theory of Sentential Unity

i. The unity of sentences involving general terms (referring to concepts) and singular terms stems from the general terms (and the concepts they refer to) being unsaturated, them being saturated at their argument positions by singular terms (respectively the concepts being saturated by the referents of the singular terms).
ii. Singular terms of the form “the concept ___” refer to concepts.
iii. Each sentence/thought has exactly one constitutive general term/concept.
iv. The saturation of the thought depends only on the saturation of the constitutive concept (i.e. a concept referred to by a singular term in argument position can stay unsaturated).

This theory differs from Frege both in having singular terms refer to concepts – as does (SU) already – and in dropping the hierarchy of concepts. Concepts now can be applied to themselves, and “( ) is categorically distinct from ( )*” can be applied to state that the concept of categorical distinction is distinct from any object. We may even allow a common domain then, containing both objects and concepts, with quantifiers running over both of them, which – finally – allows us to express (2).[5]

The downside of this vanishing of hierarchies and self-application of concepts are, of course, the ensuing contradictions. A general category of concepts and corresponding general terms immediately yields the “heterological”-paradox. The distinction between objects and concepts, being expressed thus, then requires a move to paraconsistency – i.e. to an approach supposedly

5 This resembles Russell's early use of the neutral category 'term' in his *Princples of Mathematics*.

anathema to Frege and his followers.

*Conclusion*

Frege's ontological distinction can be expressed if we commit us either to a hierarchy of basic concepts covering categorical distinction or to a paraconsistent non-hierarchical universal language. Both approaches may amend Frege's theory in allowing for some singular terms referring to concepts, as this does not endanger Frege's theory of sentential unity.

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